

Gift Wrapping

Gift-Wrap(P, n)

/* P = set of n points */

/* store vertices of CHull(P) in clockwise order in array A */

1. $A[0] \leftarrow$ leftmost (lowest) point in P;
2. $k \leftarrow 1$;
3. **repeat**
4. $A[k] \leftarrow$ first point in clockwise order around $A[k-1]$;
5. (Break ties by choosing point farthest from $A[k-1]$.);
6. **until** ($A[k] = A[0]$)

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First clockwise point

First-Clockwise(P, n)

/* P = set of n points */

/* P[0] is a vertex of CHull(P) */

/* return first point in clockwise order around P[0] */

1. $j \leftarrow 1$;
2. **for** $i \leftarrow 1$ to $n-1$ **do**
3. **if** ($\text{Orient}(P[j], P[0], P[i]) = \text{Right}$) **then**
4. $j \leftarrow i$;
5. **else if** ($\text{Orient}(P[j], P[0], P[i]) = 0$) **then**
 /* Points $P[j], P[0], P[i]$ are collinear */
6. **if** ($(P[0].x \leq P[j].x < P[i].x)$ **or**
7. $(P[0].x \geq P[j].x > P[i].x)$ **or**
8. $(P[0].y \leq P[j].y < P[i].y)$ **or**
9. $(P[0].y \geq P[j].y > P[i].y)$) **then**
10. $j \leftarrow i$;
11. **return**(j);

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Graham Scan

Graham-Scan-Upper(P, n)

/* P = set of n points */

/* store upper vertices of CHull(P) in clockwise order in array A */

1. Sort points by increasing x-coordinate.
(Break ties by increasing y-coordinate.)
Let p_0, p_1, \dots, p_{n-1} be points in sorted order.
2. $A[0] \leftarrow p_0$;
3. $A[1] \leftarrow p_1$;
4. $k \leftarrow 1$;
5. **for** $i \leftarrow 2$ to $n-1$ **do**
6. **while** ($k > 0$ **and** $\text{Orient}(A[k-1], A[k], p_i) \neq \text{Right}$) **do**
7. $k \leftarrow k-1$;
8. **endwhile**
9. $k \leftarrow k+1$;
10. $A[k] \leftarrow p_i$;
11. **endfor**

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Convex Hull Algorithms

$n = \#$ input points

$h = \#$ convex hull vertices

1. Graham scan. $O(n \log n)$
2. Gift wrapping. $O(n h)$
3. Quickhull. $O(n^2)$
4. Divide & conquer. $O(n \log n)$
5. Kirk. & Seidel. - Prune & search. $O(n \log h)$
6. Randomized incremental. $O(n \log n)$ expected.

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