An efficient computation of handle and tunnel loops via Reeb graphs

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Department of Computer Science and Engineering
The Ohio State University

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SIGGRAPH2013

Computing Handle-Tunnel Loops

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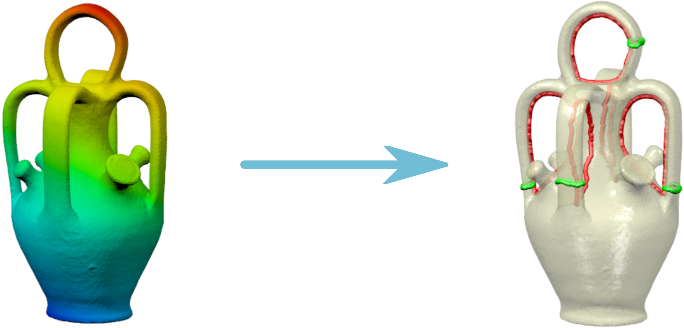
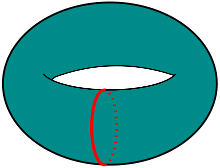


Handle and Tunnel Loops

- A *loop* $\gamma : \mathbb{S}^1 \to X$



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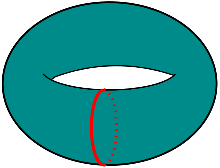

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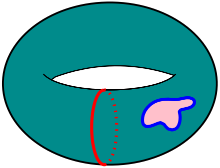

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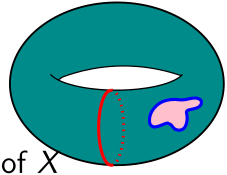

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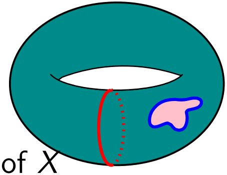

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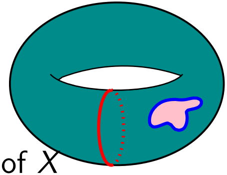

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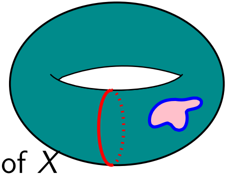

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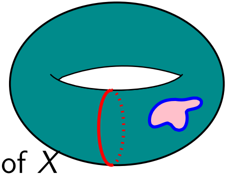

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  - partition $\mathbb{R}^3$ into : interior $\mathbb{I}$ and exterior $\mathbb{O}$;
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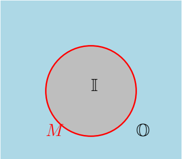

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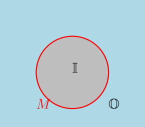

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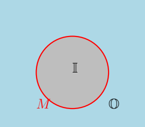

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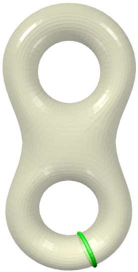

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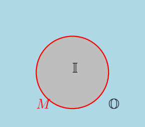

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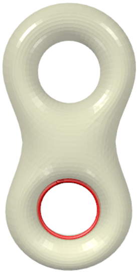

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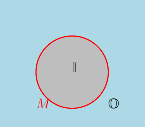

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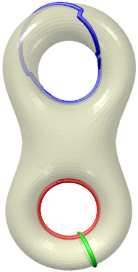

Handle and Tunnel Loops

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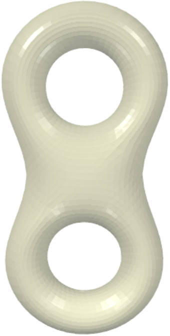

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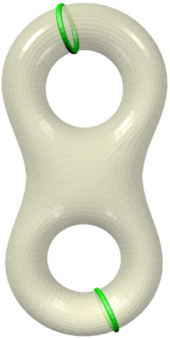

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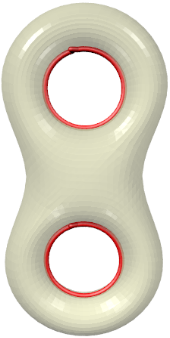

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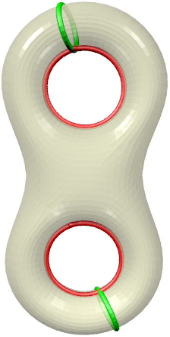

Previous methods

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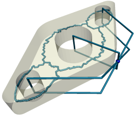

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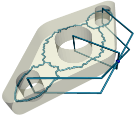

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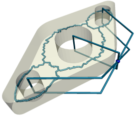

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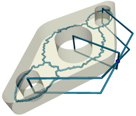

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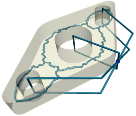

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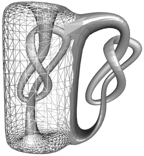

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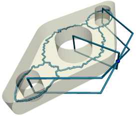

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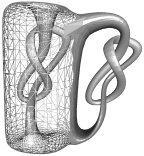

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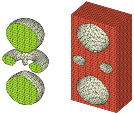

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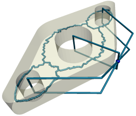

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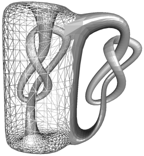

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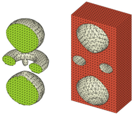

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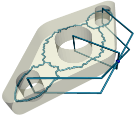

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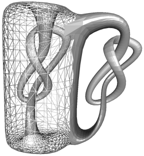

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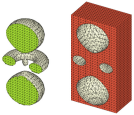

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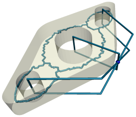

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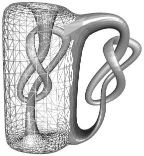

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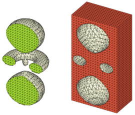

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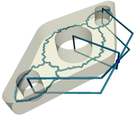

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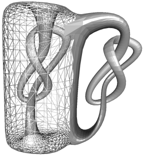

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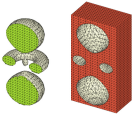

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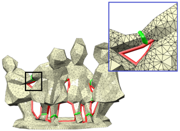

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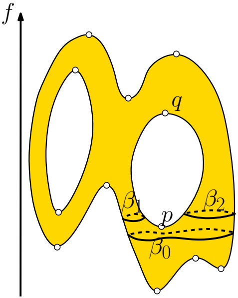

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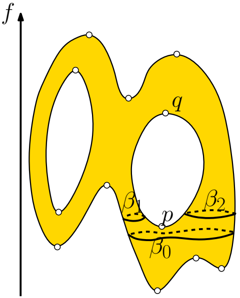

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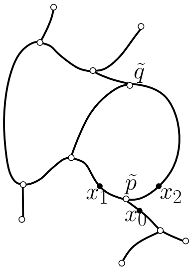

Reeb graph $\mathrm{R}_f(M)$

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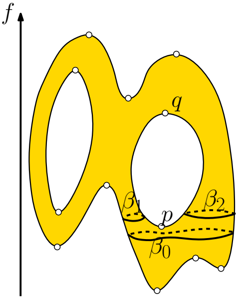

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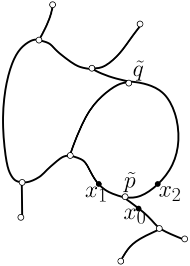
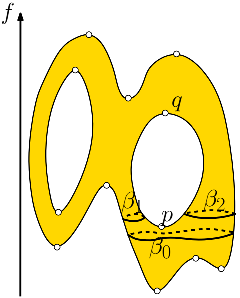


Reeb graph $\mathrm{R}_f(M)$

▷ M connected closed orientable 2-manifold with genus g



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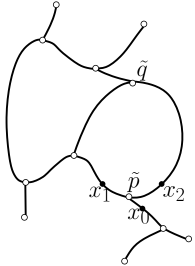
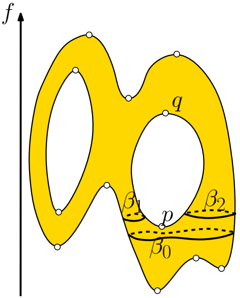


Reeb graph $\mathrm{R}_f(M)$

- M connected closed orientable 2-manifold with genus g
- f Morse function over M

Reeb Graph



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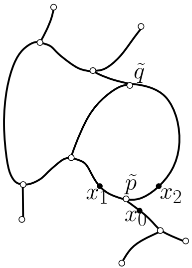

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$\Rightarrow$ $\mathrm{R}_f(M)$ has g loops ([CEHNP03])

Linking Number

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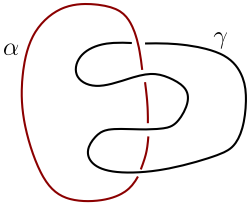
- Linking number of two disjoint loops, $\mathrm{Lk}(\alpha, \gamma)$



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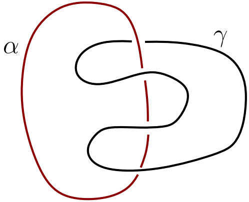
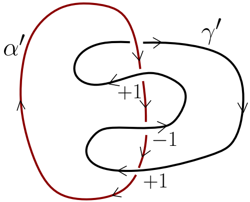


- Computation of linking number via projection;

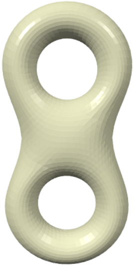


Algorithm

SIGGRAPH2013

Input A closed triangular mesh $M \subset \mathbb{R}^3$ with genus g;

1. Compute the Reeb graph Rb_M for a height function h;
2. Compute g cycles in Rb_M by a maximum spanning tree;
3. Map g cycles back to M, γ_i and compute dual level set loops $\overline{\gamma}_i$;
4. Perturb γ_i and $\overline{\gamma}_i$ to obtain α_i and $\overline{\alpha}_i$, split $\{\alpha_i, \overline{\alpha}_i\}$ into bases of $H_1(\mathbb{O})$ and $H_1(\mathbb{I})$;
5. Obtain initial handle basis h_i and tunnel basis t_i as linear combinations of γ_i and $\overline{\gamma}_i$;
6. Shorten h_i and t_i to get geometrically relevant loops;



Algorithm

SIGGRAPH2013

Input A closed triangular mesh $M \subset \mathbb{R}^3$ with genus g;

1. Compute the Reeb graph Rb_M for a random height function h;
2. Compute g cycles in Rb_M by a maximum spanning tree;
3. Map g cycles back to M, γ_i and compute dual level set loops $\overline{\gamma}_i$;
4. Perturb γ_i and $\overline{\gamma}_i$ to obtain α_i and $\overline{\alpha}_i$, split $\{\alpha_i, \overline{\alpha}_i\}$ into bases of $\mathrm{H}_1(\mathbb{O})$ and $\mathrm{H}_1(\mathbb{I})$;
5. Obtain initial handle basis h_i and tunnel basis t_i as linear combinations of γ_i and $\overline{\gamma}_i$;
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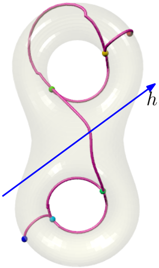

SIGGRAPH2013

Input A closed triangular mesh $M \subset \mathbb{R}^3$ with genus g;

1. Compute the Reeb graph Rb_M for a height function h;
2. Compute g cycles in Rb_M by a maximum spanning tree;
3. Map g cycles back to M, γ_i and compute dual level set loops $\overline{\gamma}_i$;
4. Perturb γ_i and $\overline{\gamma}_i$ to obtain α_i and $\overline{\alpha}_i$, split $\{\alpha_i, \overline{\alpha}_i\}$ into bases of $H_1(\mathbb{O})$ and $H_1(\mathbb{I})$;
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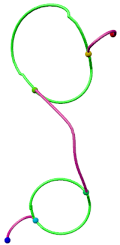

Algorithm

SIGGRAPH2013

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1. Compute the Reeb graph Rb_M for a height function h;
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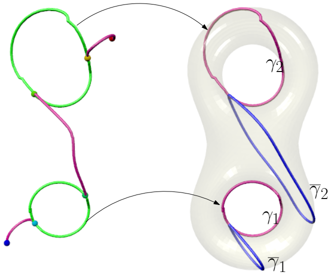

Algorithm

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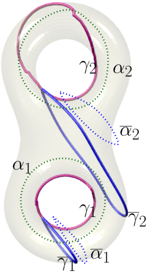

Algorithm

SIGGRAPH2013

Input A closed triangular mesh $M \subset \mathbb{R}^3$ with genus g;

1. Compute the Reeb graph Rb_M for a height function h;
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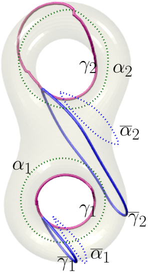
5. - Computing linking numbers $\mathrm{Lk}(\gamma, \alpha)$'s;
6. Shorten h_i and t_i to get geometrically relevant loops;



SIGGRAPH2013

Input A closed triangular mesh $M \subset \mathbb{R}^3$ with genus g;

1. Compute the Reeb graph Rb_M for a height function h;
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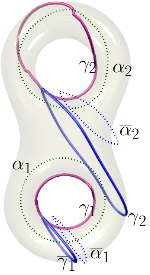
5.
   - Computing linking numbers $\mathrm{Lk}(\gamma, \alpha)$'s;
     Obtain initial handle loops h_i and tunnel loops t_i as linear combinations of γ_i and $\overline{\gamma}_i$;
6. Shorten h_i and t_i to get geometrically relevant loops;

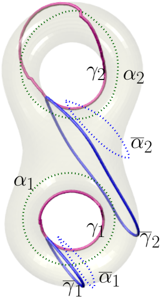


Linking number matrix

SIGGRAPH2013

$$\begin{array}{c} \\ \gamma_1 \\ \gamma_2 \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{array} \begin{array}{c} \begin{array}{cccc} \bar{\alpha}_1 & \bar{\alpha}_2 & \alpha_1 & \alpha_2 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

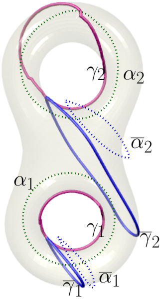
- Non-singular;



Linking number matrix

$$
\begin{array}{l} \\ \gamma_1 \\ \gamma_2 + \bar{\gamma}_2 \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{array}
\begin{array}{c}
\begin{array}{cccc} \bar{\alpha}_1 & \bar{\alpha}_2 & \alpha_1 & \alpha_2 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1+1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{array}
$$

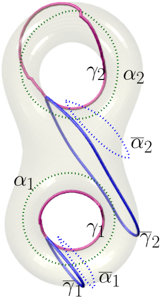
- Non-singular;



Linking number matrix

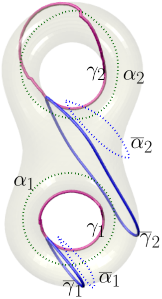
$$
\begin{array}{l} \\ \gamma_1 \\ \gamma_2 + \bar{\gamma}_2 \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{array}
\begin{array}{c}
\begin{array}{cccc} \bar{\alpha}_1 & \bar{\alpha}_2 & \alpha_1 & \alpha_2 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{array}
$$

- Non-singular;



Linking number matrix

	$\bar{\alpha}_1$	$\bar{\alpha}_2$	α_1	α_2
γ_1	1	0	0	0
$\gamma_2 + \bar{\gamma}_2$	0	1	0	0
$\bar{\gamma}_1$	0	0	1	0
$\bar{\gamma}_2$	0	0	0	1



- Non-singular;

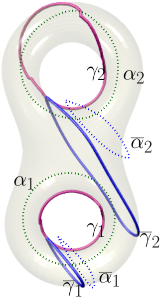
Lemma 3.3

If a loop β has zero linking number with every loop in the cycle basis of $H_1(\mathbb{I})$,

Linking number matrix

SIGGRAPH2013

$$\begin{array}{l} \\ \gamma_1 \\ \gamma_2+\bar{\gamma}_2 \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{array} \begin{array}{c} \begin{array}{cccc} \bar{\alpha}_1 & \bar{\alpha}_2 & \alpha_1 & \alpha_2 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$



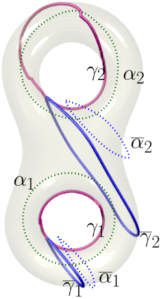
- Non-singular;

Lemma 3.3

If a loop β has zero linking number with every loop in the cycle basis of $H_1(\mathbb{I})$, but has a non-zero linking number with at least one loop in the cycle basis of $H_1(\mathbb{O})$,

Linking number matrix

	$\bar{\alpha}_1$	$\bar{\alpha}_2$	α_1	α_2
γ_1	1	0	0	0
$\gamma_2 + \bar{\gamma}_2$	0	1	0	0
$\bar{\gamma}_1$	0	0	1	0
$\bar{\gamma}_2$	0	0	0	1



- Non-singular;

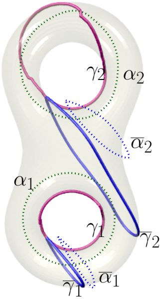
Lemma 3.3

If a loop β has zero linking number with every loop in the cycle basis of $\mathsf{H}_1(\mathbb{I})$, but has a non-zero linking number with at least one loop in the cycle basis of $\mathsf{H}_1(\mathbb{O})$, then β is a tunnel loop;

Linking number matrix

SIGGRAPH2013

$$
\begin{array}{l} \\ \gamma_1 \\ \gamma_2 + \bar{\gamma}_2 \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{array}
\begin{array}{c} \begin{array}{cccc} \bar{\alpha}_1 & \bar{\alpha}_2 & \alpha_1 & \alpha_2 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}
$$



- Non-singular;

Lemma 3.3

If a loop β has zero linking number with every loop in the cycle basis of $H_1(\mathbb{I})$, but has a non-zero linking number with at least one loop in the cycle basis of $H_1(\mathbb{O})$, then β is a tunnel loop; Symmetric for a handle loop;

Algorithm

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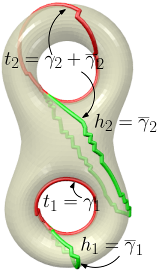
6. Shorten h_i and t_i to get geometrically relevant loops;



Algorithm

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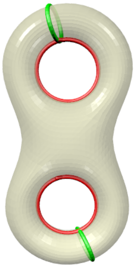
6. Shorten h_i and t_i to get geometrically relevant loops ([BCCDW12]);



Comparison

SIGGRAPH2013

- Comparison with previous method ([DLSC08]) : tessellate $\mathbb{I}$ and $\mathbb{O}$ and use persistent homology;

Model detail			Our Algorithm Timing (sec)				DLSC (sec)	
Name	Size (#Ver, #Tri),	Genus	Reeb Graph	Step 2–5	Tightening	Total	Pre-process	Loop compl
KNOTTY-CUP	($5.4K$, $10.8K$)	2	0.03	0.03	1.11	1.17	12.1	3.01
CASTING	($20K$, $40.8K$)	9	0.23	0.08	1.7	2.01	99.8	13.7
BOTIJO	($33.7K$, $67.4K$)	5	0.52	0.2	2.08	2.8	166.1	40.1
BUDDHA	($54K$, $108K$)	9	0.78	0.12	4.96	5.86	697.3	Fail
FUSEE	($121K$, $243K$)	18	2.9	0.37	40.19	43.46	1713.5	559.7
GEARBOX	($238K$, $477K$)	78	4.64	53.95	340.64	399.23	N/A	
HEPTOROID	($287K$, $573K$)	22	4.04	3.06	118.27	125.37	8797.1	2980.0
COLON	($427K$, $854K$)	160	6.38	39.47	2790.41	2836.26	N/A	
FILIGREE	($514K$, $1.03M$)	65	79.97	25.17	559.12	664.26	N/A	

Comparison

- Comparison with previous method ([DLSC08]) : tessellate $\mathbb{I}$ and $\mathbb{O}$ and use persistent homology;

Model detail			Our Algorithm Timing (sec)				DLSC (sec)	
Name	Size (#Ver, #Tri),	Genus	Reeb Graph	Step 2–5	Tightening	Total	Pre-process	Loop compl
KNOTTY-CUP	(5.4K, 10.8K)	2	0.03	0.03	1.11	1.17	12.1	3.01
CASTING	(20K, 40.8K)	9	0.23	0.08	1.7	2.01	99.8	13.7
BOTIJO	(33.7K, 67.4K)	5	0.52	0.2	2.08	2.8	166.1	40.1
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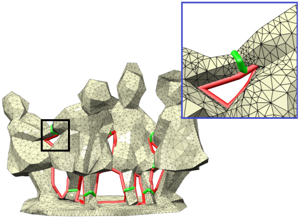
- Mesh refinement problem:



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- Comparison with previous method ([DLSC08]) : tessellate $\mathbb{I}$ and $\mathbb{O}$ and use persistent homology;

Model detail			Our Algorithm Timing (sec)				DLSC (sec)	
Name	Size (#Ver, #Tri),	Genus	Reeb Graph	Step 2–5	Tightening	Total	Pre-process	Loop compl
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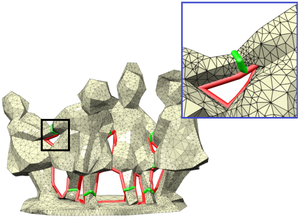
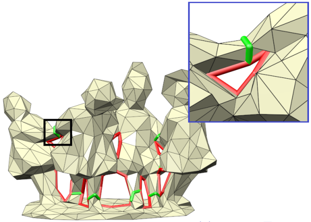
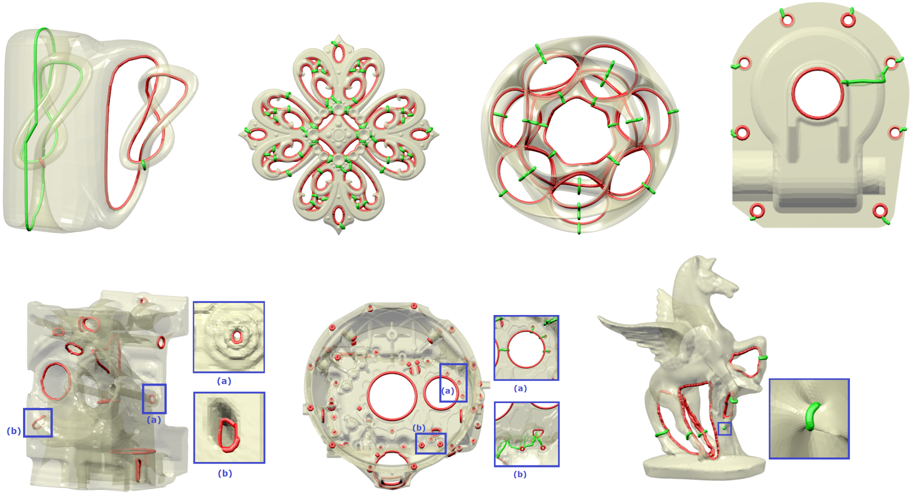


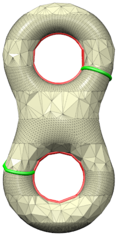


Results

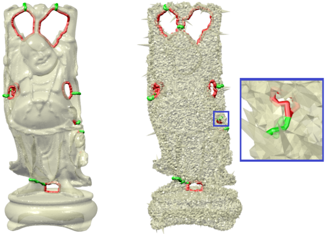
- Software available
  http://www.cse.ohio-state.edu/~tamaldey/handle/hantun.html



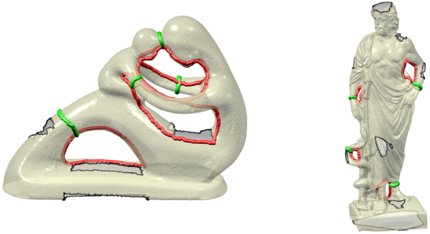
Imperfect input



Non-uniform mesh



Noisy mesh



Mesh with boundaries

THANK YOU !