Lecture 20: Catmull-Clark and Loop Subdivision [1]

Catmull-Clark Subdivision

Catmull-Clark subdivision is a generalization of bicubic spline surface subdivision applied to arbitrary mesh topology.

This method, like the bicubic B-spline subdivision, produces three types of points, face points, edge points and vertex points.

Given a face F with vertices $v_1, v_2, ..., v_n$, the new face point v_F is computed as the centroid:

$$v_F = \Sigma_{i=1}^{n} \frac{1}{n} v_i$$

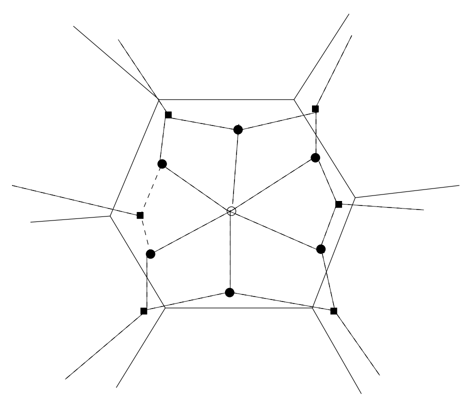
Given an edge E with endpoints v and w and adjacent faces F_1 and F_2 the new edge point v_E is the average of the four points v, w, v_{F_1}, v_{F_2}, that is,

$$v_E = \frac{v + w + v_{F_1} + v_{F_2}}{4}$$

The computation of new vertex point is a bit more complicated. Given a vertex v, if Q is the average of the new face points for all faces adjacent to v and R is the average of the midpoints of the n edges incident with v, the new vertex point v' is given as:

$$v' = \frac{1}{n} Q + \frac{2}{n} R + \frac{n-3}{n} v$$

New faces are created by connecting these new points as follows: each new face point v_F is connected to the new edge points for the edges in the boundary of F; each new vertex point v' is connected to the new edge points created for the edges incident with the vertex v.



Observations

We observe that each new face is rectangular with two edge points, one vertex point and one face point. The face points corresponding to the nonrectangular faces have degree other than four. Also, the vertex points corresponding to the vertices of degree other than four have degree other

[1] Note by Tamal K. Dey, Ohio State U.

than four. Since we produce only rectangular faces after first iteration, the number of vertices with degree other than four remain constant from second iteration onward. These vertices converges to the extraordinary points.

An older version of the rule for vertex points was:

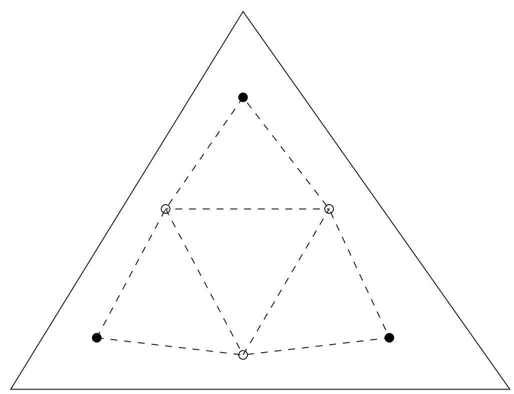
$$v' = \frac{1}{4}Q + \frac{1}{2}R + \frac{1}{4}v$$

It was observed that this surface becomes too "pointy" at places. Doo Sabin studied the scheme

$$v' = \frac{1}{n}Q + \frac{1}{n}R + \frac{n-2}{n}v$$

which gave a better result. They showed the tangent-plane continuity of this surface. Various schemes for Catmull-Clark subdivision have been proposed and analyzed from various continuity requirements.

The boundaries can also be handled in the Catmull-Clark scheme. If p_1, p_2, p_3 are three consecutive control points on a boundary curve, the two new points are generated as $p' = \frac{1}{2}p_1 + \frac{1}{2}p_2$ and $p'' = \frac{1}{8}p_1 + \frac{6}{8}p_2 + \frac{1}{8}p_3$.



Loop subdivision

Unlike other subdivision methods, Loop's method applies only to meshes with triangular faces. It splits each triangle into four triangular faces using new edge points and vertex points. Loop's scheme correspond to subdividing quartic box splines which require a control net of triangular faces with vertex degree 6.

For every edge rs that is adjacent to two triangles prs and qrs we compute the new edge point e_{rs} as

$$e_{rs} = \frac{1}{8}p + \frac{3}{8}r + \frac{3}{8}s + \frac{1}{8}q$$

This also corresponds to computing the convex combination of the three points: the centroids of the two triangles prs and qrs and the midpoint of the edge rs with weights $\frac{3}{8}$, $\frac{3}{8}$ and $\frac{2}{8}$ respectively.

For any vertex v of degree n, if $p_0, p_1, ..., p_{n-1}$ are the other endpoints of the edges incident to v, the new vertex point v' is given by

$$v' = (1 - \alpha_n)\left(\Sigma_{i=0}^{n-1}\frac{1}{n}p_i\right) + \alpha_n v$$

where α_n is a coefficient dependent on n.

Observations

We can observe that all new vertices have degree 6, except for vertices corresponding to the old vertices with degree different than 6. Thus, their number remain constant from second iteration onward. These vertices are extraordinary points. Loop observed that $\alpha_n = \frac{5}{8}$ gives good results, but tangent-plane continuity is lost at the extraordinary points. The limit surface is C^2-continuous except at extraordinary points.