

Lecture 15: *B*-spline Surface I ¹

Polynomial

B-spline surface is the generalization of *B*-spline curves. The polynomial that defines a *B*-spline surface has the following general form:

$$\mathbf{p}(u, w) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{p}_{ij} N_{i,D}(u) N_{j,R}(w)$$

The \mathbf{p}_{ij} are the $(m + 1) \times (n + 1)$ array of control points. The $N_{i,D}(u)$ and $N_{j,R}(w)$ are the basis functions, and they are same as for curves. The degree of the two corresponding curves are $D - 1$ and $R - 1$ respectively.

Matrix Form

A unit square on the parametric variables u and w is used to compute patches on the *B*-spline surface. The general matrix equation of an open uniform surface patch is:

$$\mathbf{p}_{st}(u, w) = \mathbf{U} \mathbf{P}_{DR} \mathbf{M}' \mathbf{W}^T$$

Here $s \in [1 : m + 2 - D]$, $t \in [1 : n + 2 - R]$ and $u, w \in [0, 1]$. The elements of \mathbf{P}_{DR} depends on the particular patch to be computed.

$$\mathbf{P}_{DR} = \mathbf{p}_{ij}, \quad i \in [s - 1 : s + D - 2] \quad j \in [t - 1 : t + R - 2]$$

The matrices \mathbf{M} and \mathbf{M}' are identical to the transformation matrix for the *B*-spline curve.

Idealized diagram

It is sometimes convenient to show the boundaries of the control point array and the surface patches simultaneously. See the figures 9.1, 9.2 and 9.3 in the book.

¹Note by Tamal K. Dey, Ohio State U.