Lecture 14: de Casteljau on Bézier Surface [1]

de Casteljau

We can apply the de Casteljau algorithm on surfaces to compute a point on the surface. Suppose that we have a control net with $m + 1 \times n + 1$ array of control points. We wish to compute the point $\mathbf{p}(\bar{u}, \bar{w})$.

The algorithm first applies the de Casteljau algorithm to the control points in each row $\mathbf{p}_{i0}, ..., \mathbf{p}_{in}$. For each such row it computes the point $\mathbf{p}_{i*}$. One can observe that $\mathbf{p}_{i*}$ is generated on the Bézier curve generated with the control points $\mathbf{p}_{i0}, ..., \mathbf{p}_{in}$.

The definition of $\mathbf{p}_{i*}$ is given as follows:

For every i with $0 \le i \le m$, we first compute the points $\mathbf{p}^j_{i*,k}$, where $\mathbf{p}^0_{i*,j} = \mathbf{p}_{ij}$ and

$$\mathbf{p}^j_{i*,k} = (1 - \bar{w})\mathbf{p}^{j-1}_{i*,k} + \bar{w}\mathbf{p}^{j-1}_{i*,k+1}$$

with $1 \le j \le n$ and $0 \le k \le n - j$, and we let $\mathbf{p}_{i*} = \mathbf{p}^n_{i*,0}$.

Next we apply de Casteljau algorithm on the points $\mathbf{p}_{0*}, ..., \mathbf{p}_{m*}$. For this we compute the points $\mathbf{p}^j_{i*}$, where $\mathbf{p}^0_{i*} = \mathbf{p}_{i*}$, and

$$\mathbf{p}^j_{i*} = (1 - \bar{u})\mathbf{p}^{j-1}_{i*} + \bar{u}\mathbf{p}^{j-1}_{(i+1)*}$$

with $1 \le j \le m$ and $0 \le i \le m - j$.

Finally, we will have $\mathbf{p}(\bar{u}, \bar{w}) = \mathbf{p}^m_{0*}$.

Alternatively, we could have computed $\mathbf{p}_{*0}, ..., \mathbf{p}_{*n}$ first where $\mathbf{p}_{*j}$ is obtained by de Casteljau algorithm on the control points $\mathbf{p}_{0j}, ..., \mathbf{p}_{mj}$ and then compute $\mathbf{p}^n_{*0}$ by applying the de Casteljau to the control points $\mathbf{p}_{*0}, ..., \mathbf{p}_{*n}$. We will have $\mathbf{p}^m_{0*} = \mathbf{p}^n_{*0}$.

CastelijauSurf($\mathbf{P}$)
    for $i := 0$ to m do
        for $j := 0$ to n do
            $\mathbf{p}^0_{i*,j} := \mathbf{p}_{ij}$
        endfor
        for $j := 1$ to n do
            for $k := 0$ to $n - j$ do
                $\mathbf{p}^j_{i*,k} := (1 - \bar{w})\mathbf{p}^{j-1}_{i*,k} + \bar{w}\mathbf{p}^{j-1}_{i*,k+1}$
            endfor
        endfor
        $\mathbf{p}_{i*} := \mathbf{p}^n_{i*,0}$
    endfor;
    for $i := 0$ to m do
        $\mathbf{p}^0_{i*} := \mathbf{p}_{i*}$
    endfor
    for $j := 1$ to m
        for $i := 0$ to $m - j$ do
            $\mathbf{p}^j_{i*} := (1 - \bar{u})\mathbf{p}^{j-1}_{i*} + \bar{u}\mathbf{p}^{j-1}_{i+1*}$
        endfor

[1] Note by Tamal K. Dey, Ohio State U.

endfor
Return $\mathbf{p}_{0*}^{m}$
end