Lecture 13: Bézier Surface II [1]

Degree elevation

We can achieve more flexibility in modeling with the Bézier surface by increasing its degree. The degree can be elevated by adding more control points. We would like to add control points in a manner so that the original shape is not changed. This is done by equating the coeeficients of the polynomials as follows. Suppose we want to elevate the degree in the u direction. We require that:

$$\Sigma_{i=0}^{m+1}\Sigma_{j=0}^{n}\mathbf{p}_{ij}^{1,0}B_{i,m+1}(u)B_{j,n}(w) = \Sigma_{i=0}^{m}\Sigma_{j=0}^{n}\mathbf{p}_{ij}B_{i,m}(u)B_{j,n}(w)$$

where $\mathbf{p}_{ij}^{1,0}$ are the new control points indicatign the elevation of degree by one in the u direction. Multiplying the right side by $(u + (1-u))$ we get:

$$\Sigma_{i=0}^{m+1}\Sigma_{j=0}^{n}\mathbf{p}_{ij}^{1,0}(\binom{m+1}{i})u^i(1-u)^{m+1-i}B_{j,n}(w) = \Sigma_{i=0}^{m}\Sigma_{j=0}^{n}\mathbf{p}_{ij}(\binom{m}{i})[u^i(1-u)^{m+1-i}+u^{i+1}(1-u)^{m-i}]B_{j,n}(w)$$

Equating the coefficients we get:

$$\mathbf{p}_{ij}^{1,0} = \frac{i}{m+1}\mathbf{p}_{i-1,j} + (1 - \frac{i}{m+1})\mathbf{p}_{ij} \quad for\ i = 0, ..., m+1 \quad and \quad j = 0, ..., n$$

Here $\mathbf{p}_{-1,j} = 0$.

To elevate the degree in the w direction we can use the similar computation which gives:

$$\mathbf{p}_{ij}^{0,1} = \frac{j}{n+1}\mathbf{p}_{i,j-1} + (1 - \frac{j}{n+1})\mathbf{p}_{ij} \quad for \quad i = 0, ..., m \quad and \quad j = 0, ..., n+1$$

Increasing the degree in both direction we get a new set of $(m+1) \times (n+1)$ array of control points.

$$\mathbf{p}_{ij}^{11} = \begin{bmatrix} \frac{i}{m+1} & 1 - \frac{i}{m+1} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{i-1,j-1} & \mathbf{p}_{i-1,j} \\ \mathbf{p}_{i,j-1} & \mathbf{p}_{ij} \end{bmatrix} \begin{bmatrix} \frac{j}{n+1} \\ 1 - \frac{j}{n+1} \end{bmatrix}$$
$$for \quad i = 0, ..., m+1 \quad and \quad j = 0, ..., n+1$$

Composite Bézier surface

A simple technique allows us to compose Bézier patches with at least G^1 continuity. To achieve G^0 continuity we have to use the same control points for the boundary curves where the patches meet. Suppose we want to use two 4×4 patches to meet with G^1 coninuity to produce a 4×7 patch. For this we require that the boundary curve defined by $\mathbf{p}_{03}, \mathbf{p}_{13}, \mathbf{p}_{23}$ and $\mathbf{p}_{33}$ be common to both patch and the four sets of three control points $\{\mathbf{p}_{i2}, \mathbf{p}_{i3}, \mathbf{p}_{i4}\}$ must be collinear.

See the example in Figure 8.5 in the book for an illustration of a more complex compositing.

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Rational Bézier patch

The rational Bézier patch is expressed as:

$$\mathbf{p}(u,w) = \frac{\Sigma_{i=0}^{m}\Sigma_{j=0}^{n} h_{ij}\mathbf{p}_{ij}B_{im}(u)B_{jn}(w)}{\Sigma_{i=0}^{m}\Sigma_{j=0}^{n} h_{ij}B_{im}(u)B_{jn}(w)}$$

For a constant parameter value $w = w_a$ we get the each column of control points in the control polyhedron as control points dfining the rational Bézier curve

$$\mathbf{p}_i = \frac{\Sigma_{j=0}^{n} h_{ij}\mathbf{p}_{ij}B_{j,n}(w_a)}{\Sigma_{j=0}^{n} h_{ij}B_{j,n}(w_a)}$$

and the weight h_i for $\mathbf{p}_i$ is simply $h_i = \Sigma_{j=0}^{n} h_{ij}B_{j,n}(w_a)$.