

CSE780 Homework 4  
Due Friday, Feb. 04

1. Page 353:17-1 (a), (b), (c) (1st edition) Page 402: 16-1 (a), (b) (c) (2nd edition)  
Page 446: 16-1 (a), (b), (c) (3rd edition).
2. Let  $M$  be an  $m \times m$  matrix of non-negative integers. An *independent* set of elements of this matrix is a set of elements such that no two elements lie in the same row or column. We wish to choose an independent set of elements whose sum is maximized.
  - (a) Show that the following greedy algorithm does not solve this problem:
    1. while  $M \neq 0$  do
    2.  $L \leftarrow (i, j)$  where  $(i, j)$  is max element of  $M$ ;
    3. Remove row  $i$  and column  $j$  from  $M$ ;
  - (b) Show that the greedy algorithm given above does produce an independent set whose sum is at least half the sum of the optimal solution.
3. The  $k$ th Fibonacci number is defined by the recurrence
$$F_0 = 0$$
$$F_1 = 1$$
$$F_k = F_{k-1} + F_{k-2} \text{ for } k \geq 2.$$
Prove that  $F_{k+2} = \sum_{i=0}^k F_i + 1$ .  
Prove that the  $i$ th Fibonacci number satisfies the equality  $F_i = (\phi^i - \phi'^i)/\text{sqrt}5$  where  $\phi = \frac{1+\sqrt{5}}{2}$  and  $\phi' = \frac{1-\sqrt{5}}{2}$ .

(The grader will only grade a subset of these problems.)