# Assignment 3

CSE 755

Due: November 29

1. (5 pts) Describe the set of states that satisfy the assertion '$\forall x. x = 3$'. Describe the set of states that satisfy the assertion '$\forall x. x = x$'. Justify your answers.

2. (10 pts) Which of the following results (i.e., Hoare triples) are valid? Justify your answers.

   (a) `{ true } x := 2 { true }`

   (b) `{ true } x := x { false }`

   (c) `{ false } x := 2 { true }`

   (d) `{ false } x := 2 { false }`

   (e) `{ true } while true do x := 2 end { false }`

   (f) `{ true } x := x + 1 { x = x + 1 }`

   (g) `{ x = y } t := x;  x := y;  y := t { x = y }`

   (h) `{ x >= 0 } x := y { y >= 0 }`

3. (5 pts) The following axiom for the assignment is not sound:

   $$\frac{}{\texttt{\{ true \} x := e \{ x = e \}}}$$

   Find an example that demonstrates the unsoundness of the rule. Justify your answer.

4. (10 pts) For each of the following Hoare triples (i.e., results), state whether it is valid or not. Justify your answers briefly. If you cannot decide whether a triple is valid or not, explain why you are not able to decide.

   (a) $\{x = 0\}$ **while** $x < 10$ **do** $x := x - 1;$ **end** $\{x = x + 1\}$

   (b) $\{x = 0\}$ **while** $x < 10$ **do** $x := x + 1;$ **end** $\{x = x + 1\}$

   (c) $\langle x = 0 \mid$ **while** $x < 10$ **do** $x := x - 1;$ **end** $\mid x = x + 1\rangle$

   (d) $\langle x = 0 \mid$ **while** $x < 10$ **do** $x := x + 1;$ **end** $\mid x = x + 1\rangle$

5. (20 points) *Loop reversal* is a compiler optimization that reverses the order of loop iterations. For example, suppose that `b` is an array of integers with index range $[1, \ldots, 100]$. Given the following code

```
s := 0;
i := 1;
while i <= 100 do
  s := s + b[i];
  i := i + 1;
end
```

a compiler may transform it to

```
s := 0;
i := 100;
while i >= 1 do
  s := s + b[i];
  i := i - 1;
end
```

While in general such a transformation is not always valid, in this particular example it is semantics-preserving.

**Part 1.** Suppose we wanted to show that for both versions of the code, a triple $\{\ \texttt{true}\ \}\ \ldots\ \{\ \texttt{s = } \sum_{k=1}^{100} \texttt{b[}k\texttt{]}\ \}$ can be derived. Show the detailed proofs of these two triples. In all steps of the proofs, show clearly which rule you are applying, and on which statement and pre/post-conditions. You *must* show *explicitly* all implications $\alpha \Rightarrow \beta$ you have used when applying the rule of consequence.

**Part 2.** Consider again the two programs in Part 1. Find loop invariants and termination functions and prove the corresponding *total* correctness triples. You *must* show *explicitly* all implications $\alpha \Rightarrow \beta$ you have used when applying the rule of consequence. Do not worry whether expressions are well-defined — that is, you can use the "normal" assignment axiom that was defined for partial correctness.