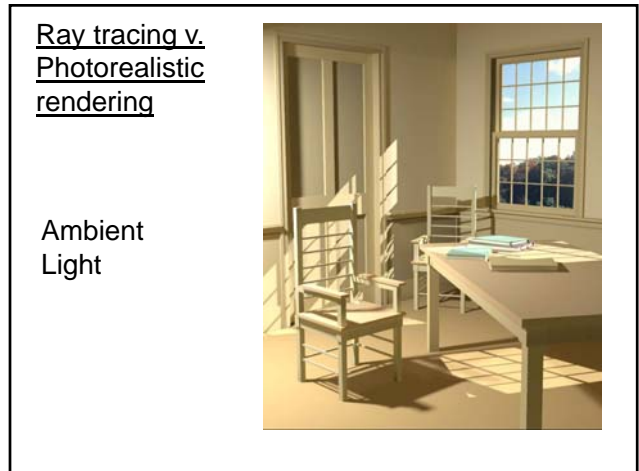


Ray tracing v. photorealistic rendering

What illumination effects are captured by ray tracing?

What illumination effects are not captured by ray tracing?



Ray tracing v. photorealistic rendering



Caustics

Ray tracing v. photorealistic rendering



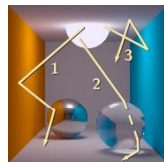
Color bleeding

Ray tracing v. photorealistic rendering

LDSE notation

- L - Light
- D - Diffuse
- S - Specular
- E - eye

- LDE
- LSE
- L[D]S]E
- LD+E
- LS'DE
- L[D]S]+E



Describe path 1/2/3 with path notation
Which is handled by ray tracing?

Rendering samples the environment

To understand how different graphics sampling strategies affect accuracy (or don't)

How can the shading / illumination be modeled mathematically?

Shading as integration

Rendering Equation (also called the Transport Equation)

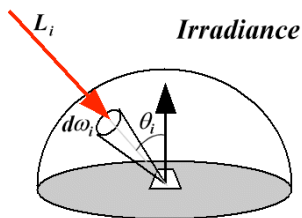
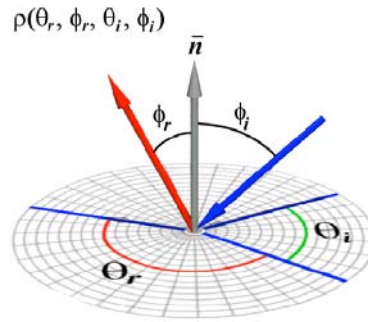
$$L_o(x, \vec{w}) = L_e(x, \vec{w}) + \int_{\Omega} f_r(x, \vec{w}', \vec{w}) L_i(x, \vec{w}') (\vec{w}' \cdot \vec{n}) d\vec{w}'$$

Bidirectional Reflectance Function (BDRF)

$$f_r(x, \vec{w}', \vec{w}) = \frac{dL_r(x, \vec{w})}{L_i(x, \vec{w}') (\vec{w}' \cdot \vec{n}) d\vec{w}'}$$

Bidirectional Reflectance Function (BDRF)

$$f_r(x, \vec{w}', \vec{w}) = \frac{dL_r(x, \vec{w})}{L_i(x, \vec{w}') (\vec{w}' \cdot \vec{n}) d\vec{w}'}$$



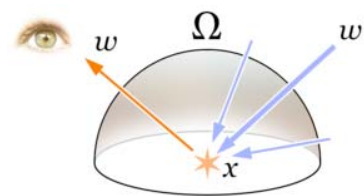
$$E_i = \int_{\Omega_i} L_i \cos \theta_i d\omega_i$$

Light at a point incoming from all directions

Radiance - light at a point in a given direction

incoming - Field Radiance

outgoing - Radiance exitance or surface radiance



Integration estimation by sampling

Shading 'equation' not analytic

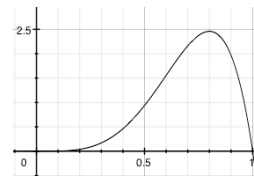
Need to estimate shading integral by sampling

Need to understand how best to control error

Basic statistics

Continuous random variable

Probability density function (pdf)



Relative likelihood for the value of a continuous variable to occur at a point

Probability of a continuous random variable falling within a set is the integral of the pdf over the set

$$p(x) \geq 0$$

$$\int_S p(x) d\mu = 1$$

Basic statistics

Expected value $E(X) = \int_{x \in S} xp(x) dx$

Standard deviation

$$\sigma(X) = \sqrt{E[(X - E(X))^2]} = \sqrt{E(X^2) - [E(X)]^2}$$

Variation

$$\text{var}(X) = E[(X - E(X))^2] = E(X^2) - [E(X)]^2$$

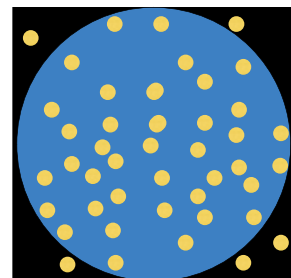
Integration estimation by sampling

Monte Carlo Integration

$$I = \int_{x \in S} f(x) dx$$

$$I \approx V \frac{1}{N} \sum_{i=1}^N f(x_i)$$

$$\text{var}(f) \equiv \sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (f(x_i) - \bar{f})^2$$



Average score by integration

(dart throwing example)

$f(x)$ - score at a point on target

$p(x)$ - pdf of score at a point on target

$$\text{average_score} = \int_S f(x)p(x)d\mu$$

$$E(x) \approx \frac{1}{N} \sum_{i=1}^N x_i \quad \text{if } x_i \text{ generated with } p(x)$$

Integration estimation by sampling

How accurate is it?

How to estimate and reduce the error?

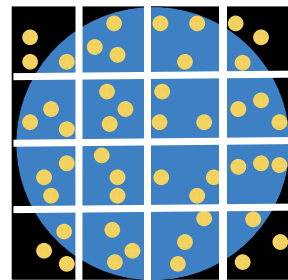
How to minimize the number of samples?

Law of large numbers

In the limit, sampling is correct

$$Prob[E(x) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N x_i] = 1$$

Stratified sampling



$$var\left(\sum_{i=1}^N \frac{g(x_i)}{p(x_i)}\right) = \sum_{i=1}^N var\left(\frac{g(x_i)}{p(x_i)}\right)$$

Sum of variances – break down into subproblems and get better estimate

$$\int_{x_i} p(x)d\mu = \frac{1}{N} \int_S p(x)\mu$$

Law of diminishing returns

error related to standard deviation

standard deviation proportional to $\frac{1}{\sqrt{N}}$

need 4x samples to get 2x better estimate

Non-uniform sampling distribution

Areas of high variability

Take more samples in those areas to reduce error

Give more weight to samples in sparse areas (value reflects more area)

Importance sampling

sample non-uniform distribution

uniform sample & weigh by distribution

non-uniform sample & weigh evenly

$$E(f(x)) = \int_{x \in S} f(x)p(x)d\mu \approx \frac{1}{N} \sum_{i=1}^N f(x_i)$$

Alternatives to Ray Tracing

Trace more rays from eye?

Trace rays from light source?

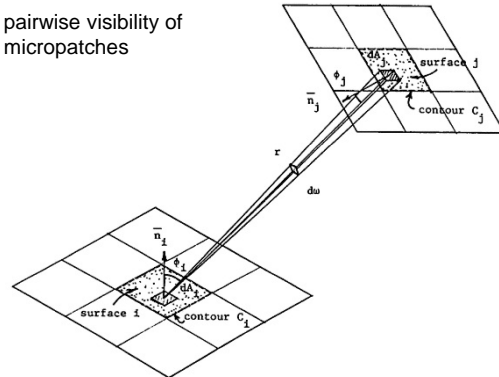
Do both?

Radiosity - Jim Kaijva

Propagate diffuse light around environment: LD'E
 Precompute diffuse (viewer independent) illumination
 From Heat Transfer Theory developed in the '50s
 Not a ray tracing approach - usually combined with it
 Break environment up into micro-patches
 Determine pairwise visibility between each pair of micro-patches

Radiosity - Form Factor

pairwise visibility of micropatches

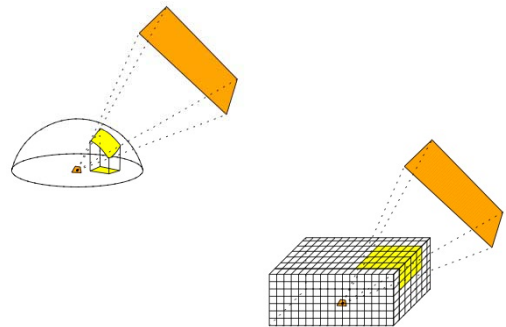


Radiosity - form factor

$$F_{12} = \frac{1}{A} \int_{x_1 \in S_1} \int_{x_2 \in S_2} \frac{g(x_1, x_2) \cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi \|x_1 - x_2\|^2}$$

$$F_{12} \approx A_2 \frac{g(x_1, x_2) \cos\theta_1 \cos\theta_2}{\pi \|x_1 - x_2\|^2}$$

Radiosity - form factor



Radiosity

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & -\rho_1 F_{1,2} & \cdots & -\rho_1 F_{1,N} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & \cdots & -\rho_2 F_{2,N} \\ \vdots & & & \vdots \\ -\rho_N F_{N,1} & -\rho_N F_{N,2} & \cdots & 1-\rho_N F_{N,N} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}$$

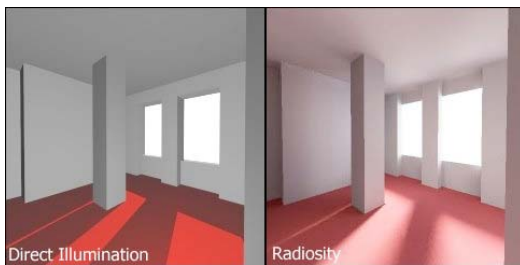
Solve large matrix equation

Radiosity - progressive refinement

$$B_i = E_i + \rho_i \sum_{j=1} B_j F_{ji}$$

Initially $B_i = E_i$

Iteratively add emissions propagated around environment

RadiosityRadiosity

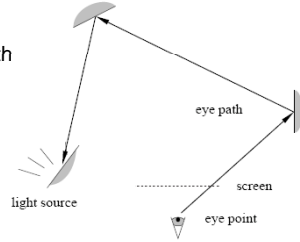
Path Tracing - Jim Kajiya

Extension of ray tracing

At every intersection, select random direction to generate secondary (and beyond) ray - Do this a lot

Until it hits a light source
Until a certain maximum depth

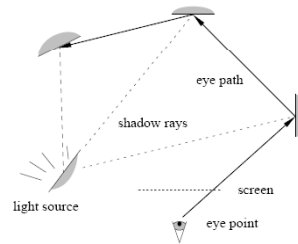
Probabilistically terminate
"Russian Roulette"



Diagrams from:
<http://graphics.ucsd.edu/~iman/BDPT/>

Path Tracing

Improve by illuminating each intermediate point



Path Tracing

Accurate - if enough rays are generated

Inefficient

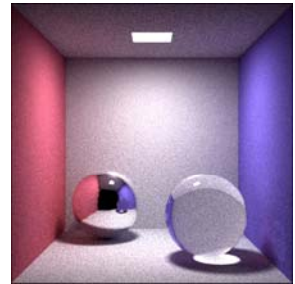
Unbiased

Accurate in the limit



http://feblemind.tuxfamily.org/dotclear/images/cimetiere-des-pixels/sunflow/sponza/sponza_SF-path_tracing-logo.png

Path Tracing



From:
<http://graphics.ucsd.edu/~iman/BDPT/>

Path Tracing



<http://sunflow.sourceforge.net/gallery/v0060/livingroom.png>

Light Tracing - Philip Dutre

Generate a bunch of 'particles' at light source

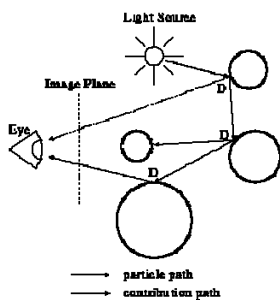
Trace each particle through environment

Every time a hit
 calculate contribution to eye
 randomly bounce particle into scene

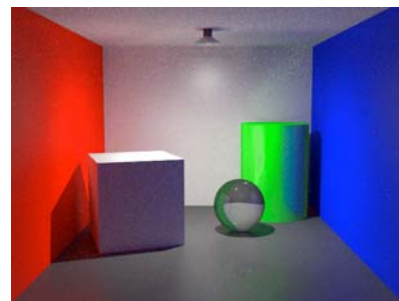
Probabilistically terminate

unbiased

Light Tracing



Light Tracing



www.graphics.cornell.edu/~eric/thesis/images.html

Light Tracing



Photon Mapping - Henrik Wann Jensen

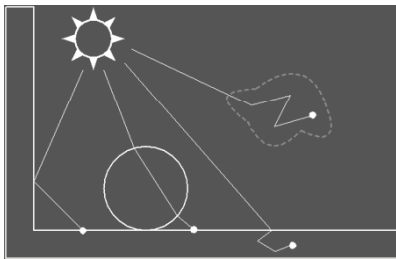
2 pass algorithm

1st pass - photons shot from light source and 'deposited' on surfaces as they bounce around the environment - probabilistically

2nd pass - ray tracing with indirect illumination coming from querying the photon map by shooting secondary rays

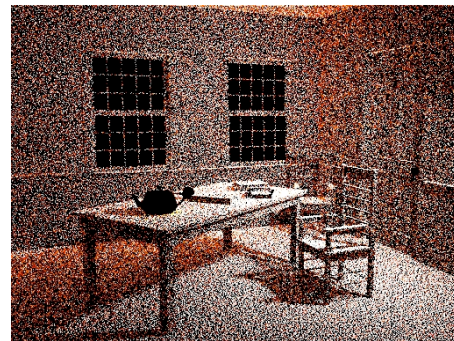
Biased - photon map is sparse, so at query point photon map is interpolated from close-by photon deposits

Photon Mapping



http://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/zackw/photon_mapping/PhotonMapping.html

Photon Map



<http://igt.akpeters.com/papers/Christensen99/Fig2a.jpg>

Photon Mapping



From

<http://www.ypoart.com/tutorials/photon/index.php>

Photon Mapping



Photon Mapping



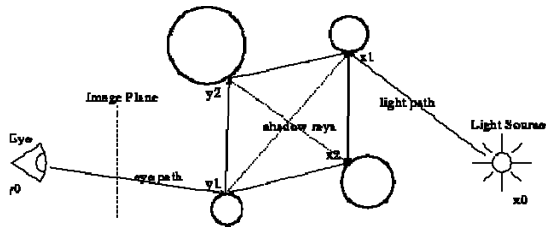
Bi-Directional Path Tracing

Generate paths from light

Generate paths from eye

Hook up vertices from one path to vertices of other path,
to get multiple paths from eye to light

Bi-Directional Path Tracing

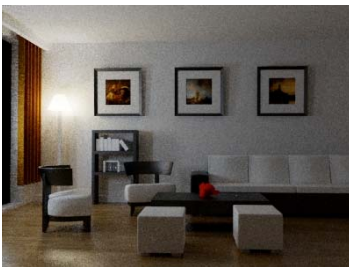


From:
<http://graphics.ucsd.edu/~iman/BDPT/>

Bi-Directional Path Tracing



Bi-Directional Path Tracing



Bi-Directional Path Tracing



Consider

