Object Intersection

Object Representation

Implicit forms
F(x,y,z) = 0 — testing

Explicit forms
Analytic form x = F(y,z) — generating

Parametric form (x,y,z) = P(t)

Ray-Object Intersection

Implicit forms
F(x,y,z) = 0

Ray: P(t) = (x,y,z) = source + t*direction = s + t*c

Solve for t: F(P(t)) = 0

Ray-Sphere Intersection

Implicit form for sphere at origin of radius 1

$$F(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$$

Ray: $P(t) = (x,y,z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z)$

Solve: …

$$F(P(t)) = (s_x + tc_x)^2 + (s_y + tc_y)^2 + (s_z + tc_z)^2 - 1 = 0$$
$$= s_x{}^2 + s_y{}^2 + s_z{}^2 + 2t(s_x c_x + s_y c_y + s_z c_z) + t^2(c_x{}^2 + c_y{}^2 + c_z{}^2) - 1 = 0$$

Use quadratic equation…

Ray-Sphere Intersection

$At^2 + Bt + C = 0$
$A = |c|^2$
$B = 2s \cdot c$
$C = |s|^2 - 1$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$B^2-4AC < 0$ => no intersection
$= 0$ => just grazes
> 0 => two hits

Axis-Aligned Cuboid
(rectangular solid, rectangular parallelpiped)

Ray equation

$P(t) = s + tc$

Planar equations

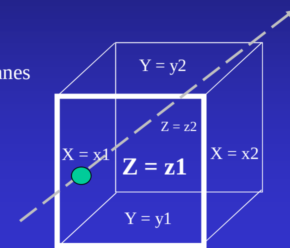
Solve for intersections with planes

$t_{x1} = (x1 - s_x)/c_x$

$t_{x2} = (x2 - s_x)/c_x$

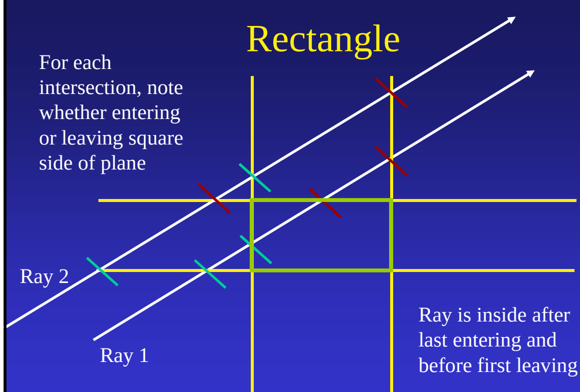
$t_{y1} = (y1 - s_x)/c_x$

…



Rectangle

For each intersection, note whether entering or leaving square side of plane

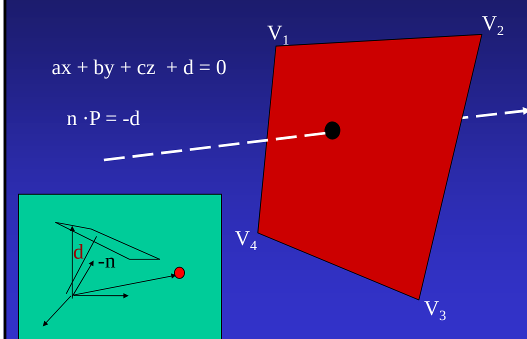


Ray is inside after last entering and before first leaving

Ray-Plane for arbitrary plane

$ax + by + cz + d = 0$

$n \cdot P = -d$



Normal Vector

Given ordered sequence of points defining a polygon how do you find a normal vector for the plane?

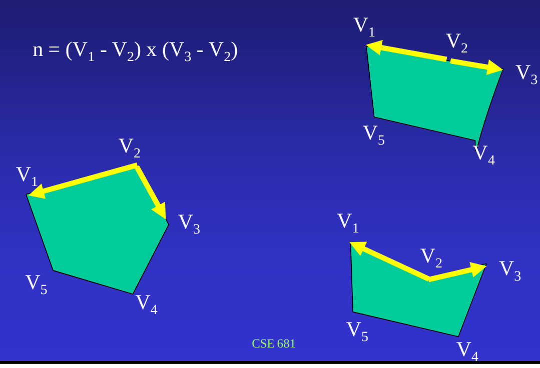
Note: 2 normal vectors to a plane, colinear and one is the negation of the other

Ordered: e.g., clockwise when viewed from the front of the face

Right hand v. left hand space

Normal Vector

$n = (V_1 - V_2) \times (V_3 - V_2)$

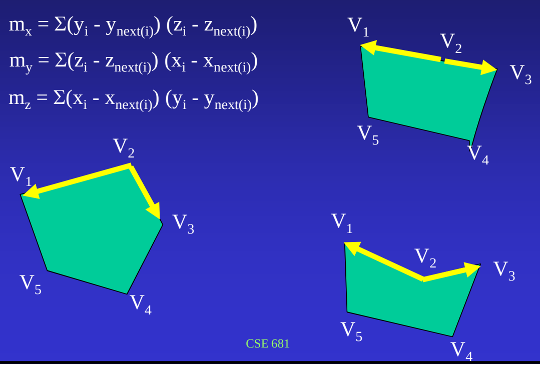


Normal Vector

$m_x = \Sigma(y_i - y_{next(i)})\,(z_i - z_{next(i)})$

$m_y = \Sigma(z_i - z_{next(i)})\,(x_i - x_{next(i)})$

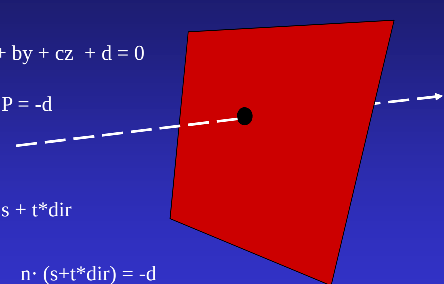
$m_z = \Sigma(x_i - x_{next(i)})\,(y_i - y_{next(i)})$



Ray-Plane

$Ax + by + cz + d = 0$

$n \cdot P = -d$



$P = s + t*dir$

$n \cdot (s+t*dir) = -d$

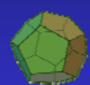
$n \cdot s + t*(n \cdot dir) = -d$

$t = -(d + n \cdot s)/(n \cdot dir)$

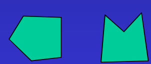
Ray-Polyhedron

Polyhedron - volume bounded by flat faces
Each face is defined by a ring of edges
Each edge is shared by 2 and only 2 faces

The polyhedron can be convex or concave



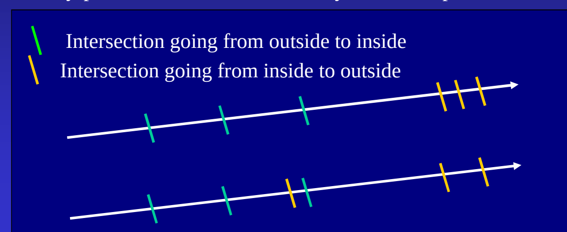
Faces can be convex or concave



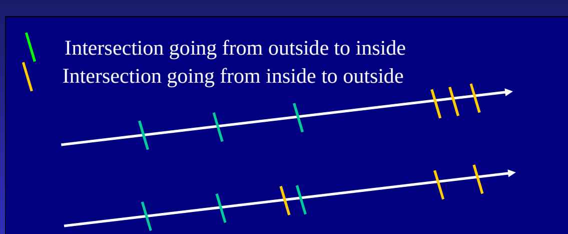
Convex Polyhedron

volume bounded by finite number of <u>infinite</u> planes

Computing intersections is similar to cube but using ray-plane intersection and arbitrary number of planes



Convex Polyhedron



Record maximum entering intersection - enterMax
Record minimum leaving intersection - leaveMin

If (enterMax < leaveMin) polyhedron is intersected

Concave Polyhedron

Find closest face (if any) intersected by ray

Need ray-face (ray-polygon) intersection test

Ray-Polyhedron

1. Intersect ray with plane

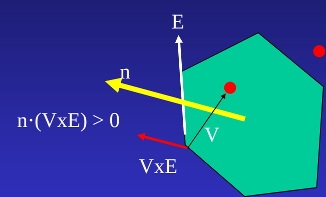
2. Determine if intersection point is inside of 2D polygon

   A) Convex polygon

   B) Concave polygon

Ray-Convex Polygon

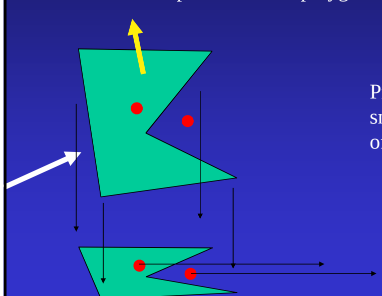
Test to see if point is on
'inside' side of each edge



Dot product of
- normal
- Cross product of
  - ordered edge
  - vector from edge source to point of intersection

Ray-concave polygon

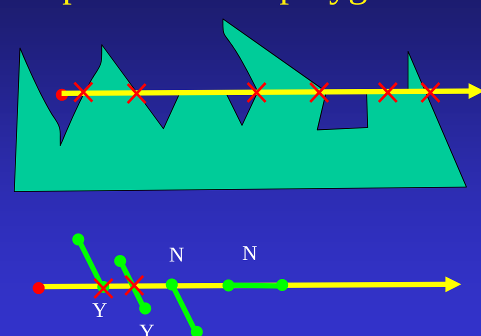
Project plane and point of intersection to 2D plane
2D point-inside-a-polygon test



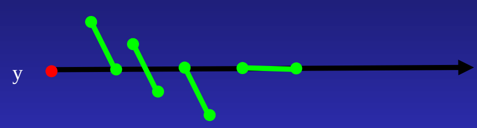
Project to plane of 2 smallest coordinates of normal vector

Form semi-infinite ray and count ray-edge intersections

2D point inside a polygon test



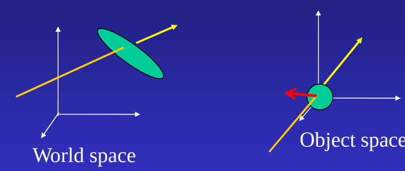
2D point inside a polygon test



$$if\ (((y < y_2)\ \&\&(y >= y_1))\ ||\ ((y < y_1)\ \&\&(y >= y_2)))$$

Transformed objects

e.g., Ellipse is transformed sphere



Intersect ray with transformed object

Use inverse of object transformation to transform ray
Intersect transformed ray with untransformed object

Transformed objects

$r(t) = s + tc$ World space ray

$s = [s_x, s_y, s_z, 1]$

$c = [c_x, c_y, c_z, 0]$

$R(t)^T = M^{-1}s^T + M^{-1}c^T$ Object space ray

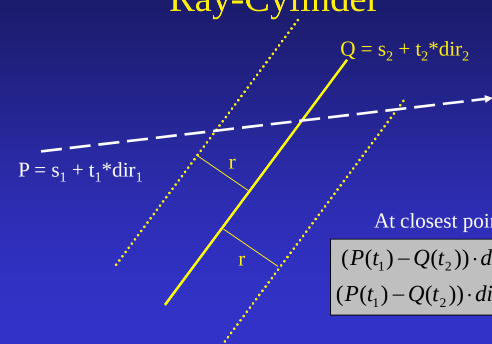
Intersect ray with object in object space

Transform intersection point and normal back to world space

$$P_{world}{}^T = MP_{object}{}^T$$

$$N_{world}{}^T = (M^{-1})^T N_{object}{}^T$$

Ray-Cylinder



At closest points

$$(P(t_1) - Q(t_2)) \cdot dir_1 = 0$$
$$(P(t_1) - Q(t_2)) \cdot dir_2 = 0$$

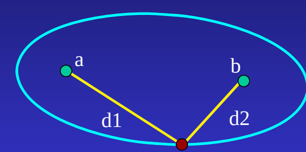
Ray-Cylinder

$$(P(t_1) - Q(t_2)) \cdot dir_1 = 0$$
$$(P(t_1) - Q(t_2)) \cdot dir_2 = 0$$

$$(s_1 + t_1 dir_1 - (s_2 + t_2 dir_2)) \cdot dir_1 = 0$$
$$(s_1 + t_1 dir_1 - (s_2 + t_2 dir_2)) \cdot dir_2 = 0$$

Ray-Ellipsoid

Geometric construction: all points p such that |p-a| + |p-b| = r



|P(t)-a| + | P(t)- b| = r

Ray-Quadric

$$P(t) = (x, y, z) = s + tc = (s_x + tc_x, s_y + tc_y, s_z + tc_z)$$

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

http://en.wikipedia.org/wiki/Quadric

Ray-Whatever

Algebraic solution v. Numeric solution

$$F(P(t)) = 0$$