

Procedural Models

- How to easily generate complex data?
- Data from parameterized procedure and/or digitized
- Data amplification
- Ray trace directly v. convert to polygonal models and use z-buffer

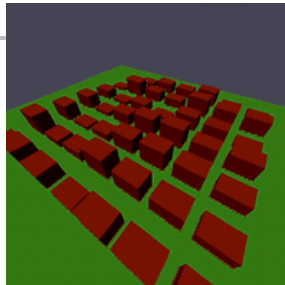
Basic Procedural Approaches

- composition from primitive shapes
- extrusion
- surface of revolution
- lofting
- sweep operator

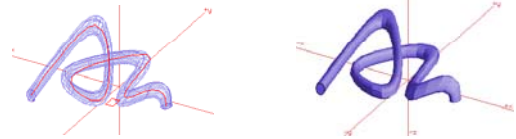
Composition

Procedurally generated

Controlled randomness



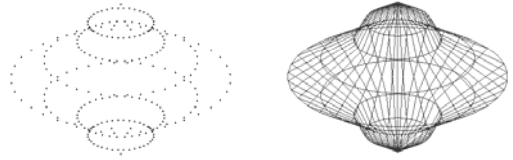
Extrusion



Surface of revolution

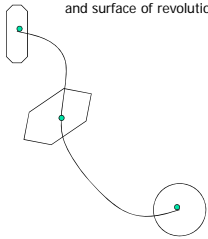


Lofting



Sweep Operator

Combines extrusion, lofting,
and surface of revolution

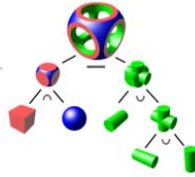


More sophisticated approaches

- CSG
- Subdivision surfaces
- Curves & surfaces
- Fractals
- Implicit functions

CSG: constructive solid geometry

- primitive shapes
- union, difference, intersection
- tree structure representation
- Ray trace directly
- Evaluate to polyhedron with Boolean operators
- OpenGL rendering using stencil buffer
- <http://www.opengl.org/resources/code/samples/advanced/advanced97/notes/node11.html>



Subdivision surfaces

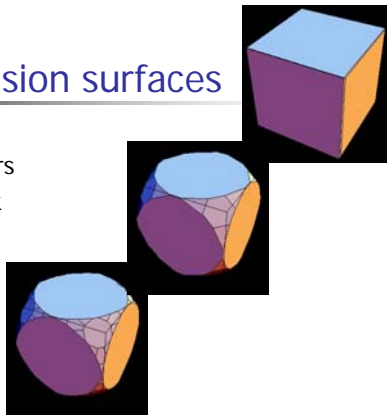
- Initial coarsely defined geometry
- Refine geometry by
 - rounding corners,
 - subdividing faces and edges,
 - smoothing faces

What is resulting limit surface?

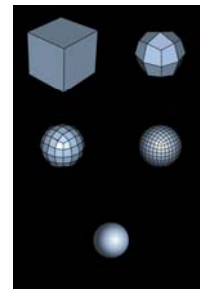
- What is continuity of limit surface?

Subdivision surfaces

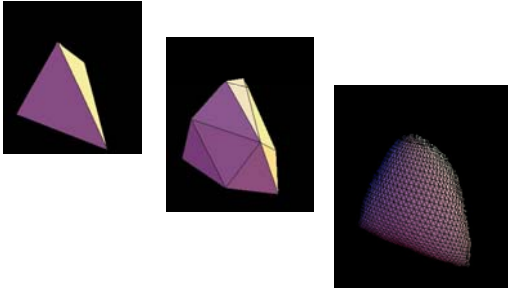
- Round corners
- Catmull-Clark
- Doo-Sabin
- Loop
- Butterfly



Catmull-Clark Subdivision



Loop Subdivision



Curved surface patches

- Hermite curve & patches
- Bezier curve & patches
- Catmull-Rom spline
- B-spline

Hermite Curve

- Given: starting and ending points and tangents
- Determine cubic equation that satisfies constraints

$$P(u) = au^3 + bu^2 + cu + d$$

$$P'(u) = 3au^2 + 2bu + c$$

$$P(0) = p0 = d$$

$$P(1) = p1 = a + b + c + d$$

$$P'(0) = t0 = c$$

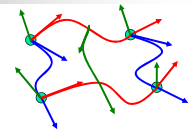
$$P'(1) = t1 = 3a + 2b + c$$

$$P(u) = UMB = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p0 \\ p1 \\ t0 \\ t1 \end{bmatrix}$$

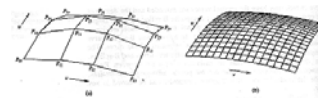


Hermite Patch

- Given: starting and ending points and tangents
- Determine bicubic equation that satisfies constraints



$$P(u, v) = uMBM^T v$$



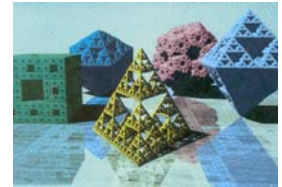
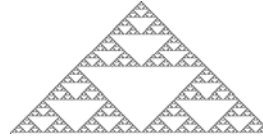
Fractal geometry

- Objects that exhibit similarity under scale
- e.g., Koch curve



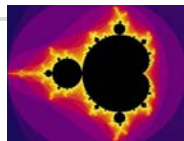
- Mathematically curious - continuous, infinite length, nowhere differentiable, bounded

Serpinski Gaskets

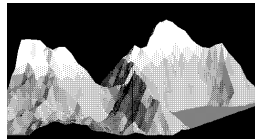
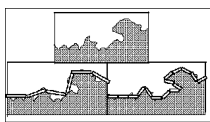


Other fractal shapes

- Mandelbrot set
- Coastlines
- Mountains



statistically self-similar under scale



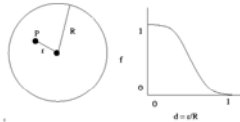
Create mountains

1. Subdivide triangle
2. Displace upward
3. Reduce amplitude of displacement
4. Recurse on subdivisions

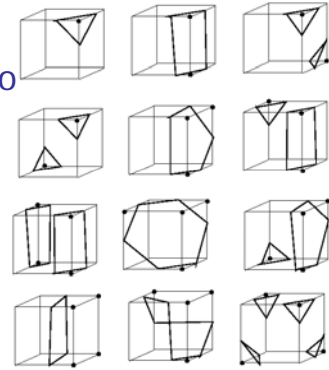


Implicit functions & isosurfaces

- $f(x,y,z) = 0$
- "density" function
- distance-based implicit functions
- Marching Cubes algorithm



Marching Cubes : convert to polygons



Implicit functions & isosurfaces

