CSE 763 **Spring 2007**
Prof. Paul Sivilotti

Assignment #2: Reasoning About Programs — Safety

DUE: in class, Friday, April 13th.

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1. **State Transition Diagrams** (4 points)

   Consider the following program, where $\mathbb{N}_i$ denotes the set of natural numbers from 0 to $i-1$ inclusive, and all arithmetic operations are modulo the appropriate range:

   Program $Q1$
   var $x : \mathbb{N}_3, y : \mathbb{N}_4$
   initially $(x \geq 1) \ \wedge \ (y = 0)$
   assign
   $x = y \ \longrightarrow \ x := x + 1$
   $[\!]$ $even.y \ \longrightarrow \ y := x + y$
   $[\!]$ $y \geq x + 2 \ \longrightarrow \ x, y := 0, 3$

   Draw a state transition diagram for this program. You may omit edges that correspond to the implicit `skip` action.

2. **Fixed Points** (6 points)

   Part I. (2 points)

   From the state transition diagram in question #1, what are the fixed points of program Q1? Characterize this list of fixed points with as simple a predicate as possible.

   Part II. (4 points)

   Calculate the fixed points of the following program, simplifying your answer as much as possible. You may assume that $M \geq 0$ and $N > 0$.

   Program *Strange*
   var $x, y, z, k : \mathbb{Z}$
   initially $x = 0 \ \wedge \ y = M \ \wedge \ z = N \ \wedge \ k = 1$
   assign
   $y \geq 2 * z \ \longrightarrow \ z, k := 2 * z, 2 * k$
   $[\!]$ $y < 2 * z \ \longrightarrow \ z, k := N, 1$
   $[\!]$ $y \geq z \ \longrightarrow \ x, y := x + k, y - z$

3. **Semantics of Actions** (10 points)

Part I. (3 points)

We defined the semantics of an action by the Assignment Axiom. That is, we stated that the triple

$$\{P\} \quad g \longrightarrow x := E \quad \{Q\}$$

can be proven by establishing:

$$[(P \wedge g \Rightarrow Q_E^x) \wedge (P \wedge \neg g \Rightarrow Q)]$$

Consider the following alternate proof obligation:

$$[P \Rightarrow (g \wedge Q_E^x) \vee (\neg g \wedge Q)]$$

Are these two expressions equivalant? If so, which formulation do you prefer and why? If not, give an example of a Hoare triple that illustrates the difference.

Part II. (5 points)

Prove or disprove the following Hoare triples:

(a) $\{P\} \quad g \longrightarrow x := E \quad \{true\}$

(b) $\{y = x + 3\} \quad y := x - 1.5 * y + 6 \quad \{x + 2 * y = 3\}$

(c) $\{x = 2 \vee x = 4\} \quad x \neq 4 \longrightarrow y := x \quad \{x = 2\}$

Part III. (2 points)

What must be true for the following Hoare triple to be true?

$$\{P\} \quad g \longrightarrow x := E \quad \{P\}$$

4. **Safety** (6 points)

Prove the following invariant for program *Strange* given in Question 2:

$$x * N + y = M \wedge y \geq 0 \wedge z = k * N \wedge k \geq 1$$

If this program reaches a fixed point, what has it computed?