CSE 763 **Spring 2007**
Prof. Paul Sivilotti

Assignment #1: Predicate Calculus

DUE: in class, Friday, April 6th.

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1. **Equivalence** (4 points)

   Consider the following expression:

   $$P_0 \equiv P_1 \equiv P_2 \equiv \ldots \equiv P_N$$

   where each P_i is a predicate and N is greater than or equal to 0.

   Given a state and the value of each P_i in that state (ie true or false), state a simple rule for determining whether or not the entire expression is true for that state. You may consider several cases, based on the value of N, although the best answer will not require such a distinction.

2. **Predicate Calculus** (12 points)

   Carefully prove or disprove the following, using only the axioms and theorems given in Chapter 1 of the notes. Once you have proven the property in one part of this question, you may use that property in proofs of other parts.

   (a) Idempotence of $\wedge$: $[X \wedge X \equiv X]$

   (b) Absorption : $[X \vee (X \wedge Y) \equiv X]$

   (c) In class, we postulated as an axiom the distribution of $\vee$ over $\equiv$:

   $$[X \vee (Y \equiv Z) \equiv X \vee Y \equiv X \vee Z]$$

   Hypothesize (and prove!) a somewhat similar rule for $\wedge$ distribution over $\equiv$. What is the surprising difference?

   (d) $\vee$ distribution over $\wedge$: $[X \vee (Y \wedge Z) \equiv (X \vee Y) \wedge (X \vee Z)]$

   (e) $[(X \Rightarrow Y) \equiv Y \equiv X \vee (\neg X \wedge Y)]$

   (f) In class, we stated the following theorem about implication: $[X \Rightarrow Y \equiv \neg X \vee Y]$. Prove this theorem, without – of course – using the theorem itself!

3. **Everywhere Brackets** (6 points)

   Leibniz's rule states, informally, that a function applied to two things that are the same, yields the same result. More formally, it can be expressed:

   $$[X \equiv Y] \Rightarrow [f.X \equiv f.Y]$$

   A very different property, known as *punctuality*, states the following:

   $$[(X \equiv Y) \Rightarrow (f.X \equiv f.Y)]$$

   Whereas Liebniz's rule can be taken as a fundamental property of equivals and function application, punctuality is a property satisfied by some functions, but not all.

(a) (2 points) Give an example of a function (from predicates to predicates) that *is* punctual. Justify your answer.

(b) (3 points) Give an example of a function (from predicates to predicates) that *is not* punctual. (Your example should be very specific about the state space and predicates in question.) Justify your answer.

(c) (1 point) Why do you think the term "punctual" is used for this property?

4. **Quantification** (2 points)

Translate the following English sentences into our quantification notation. Introduce any predicates you feel necessary and take care to preserve the meaning of the sentence.

(a) Every red ball in the bag is heavier than any green ball in the bag.

(b) All is not lost.

(c) Anything that influences nothing, does not influence itself.