CIS 780 Homework 5

Due: Friday, November 7

Exam II: Friday, November 14. Topic: dynamic programming.

For each problem you should give a complete solution, including a recurrence relation, boundary conditions, and a *nonrecursive* algorithm. You should also give the asymptotic running time of your algorithm.

1. Consider an $m \times m$ chessboard with an integer written on each square of the board. A pawn starts out in the first column and first row and advances one row at a time to the last row. To advance, the pawn may move forward one row along a column or a diagonal. Thus a pawn in row i, column j, can move to columns $j-1$, j or $j+1$ in row $i+1$. The weight of a move is given by the integer on the square to which the pawn moves. The weight of a sequence of moves is the sum of the weights of each move.

   Describe an algorithm which computes the minimum weight sequence of moves by which a pawn can advance from the first to the last row.

2. Let $A = a_1 a_2 \ldots a_m$ and $B = b_1 b_2 \ldots b_n$ be two strings of characters. We want to transform A into B using following operations:

   - delete a character
   - add a character
   - change a character

   Write a dynamic programming algorithm that finds the minimum number of operations needed to transform A into B.

3. N jobs are to be scheduled for processing on one machine. Job i, $1 \leq i \leq N$, needs t_i units of processing time. If job i is finished by time T, where T is a given deadline, then a profit p_i is earned; otherwise, a penalty q_i is imposed. (Both p_i and q_i are positive integers.) We want to select a subset S of jobs such that
   (i) $\sum_{i \in S} t_i \leq T$, and
   (ii) $f(S) = \sum_{i \in S} p_i - \sum_{i \notin S} q_i$ is maximum.
   Show how to find such a set of jobs using dynamic programming.