

# CSE 780 Homework 1

Due Friday, October 3 by class time

1. Order the following function by asymptotic dominance. That is, produce a permutation  $f_1(n), f_2(n), \dots$  such that  $f_i = O(f_{i+1})$ .

(a)  $f(n) = n! * n^2$

Hint: use Stirling's formula,  $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right)$ .

(b)  $f(n) = 2^{2^3}$

(c)  $f(n) = 1/n^2$

(d)  $f(n) = 4^{\sqrt{n}}$

(e)  $f(n) = n^{2^n}$

(f)  $f(n) = \log_2 \log_2(n^5 + n^2)$

(g)  $f(n) = 19n^{1.5} + 3n^{2.1} + \sqrt{n}$

(h)  $f(n) = 27 \log_7(n) + \sqrt{\log_2(n)}$

(i)  $f(n) = (\log_2 n)^3$

(j)  $f(n) = 5^{\log_2 n} + \sqrt{n}$

(k)  $f(n) = 2^{(2^n)}$

2. Prove or disprove the following statement: If  $f(n) = \Theta(n^2)$ , then  $f(n)$  is asymptotically monotonically nondecreasing (i.e.,  $f(n) \leq f(n+1)$  for all sufficiently large integers  $n$ ). (To disprove a statement, you need to give a specific counterexample.)
3. Prove or disprove the following statement: If  $f(n) = O(g(n))$ , then  $2^{f(n)} = O(2^{g(n)})$ . (To disprove a statement, you need to give a specific counterexample.)
4. How many dollar signs (\$) will the following procedures print? Give your answer, in  $\Theta$  notation, as a function of  $n$ . Justify your answer.

Procedure dollar( $n$ )

$j \leftarrow 1$

  while  $j < n$  do

$j \leftarrow j + j$

$k \leftarrow 2$

    while  $k < n$  do

$k \leftarrow k * k$

      print(\$)

**End of Homework 1: there were a total of four questions.**