

Shortest Paths

CSE 680

1 Single Source Shortest Path

- Problem: Given an undirected, connected, weighted graph $G(V, E)$ and a node $s \in V$, find a shortest path between s and x for each $x \in V$. (Assume positive weights.)
- Assume $V = \{1, 2, \dots, n\}$.
- Observation: a shortest path between s and v may only pass through nodes which are closer to s than is v .
- That is, if $d(v_1) \leq d(v_2) \leq d(v_3) \leq \dots \leq d(v_n)$, where $d(x)$ denotes the shortest distance between s and x , then a shortest path between s and v_k may only pass through nodes in $\{v_1, \dots, v_{k-1}\}$.
- We will compute the shortest distance between s and v_k in the order of $v_1, v_2, v_3, \dots, v_n$.

Dijkstra's Algorithm($G = (V, E), s$)

$D[s] \leftarrow 0$

$Parent[s] \leftarrow 0$

$V' \leftarrow \{s\}$

for $i \leftarrow 1$ **to** $n - 1$ **do**

 find an edge (u, v) such that $u \in V', v \notin V'$

 and $D[u] + length[u, v]$ is minimum;

$D[v] \leftarrow D[u] + length[u, v];$

$Parent[v] \leftarrow u;$

$V' \leftarrow V' \cup \{v\};$

endfor

Data Structures:

- The given graph: $length[1..n, 1..n]$.
- Shortest distances: $D[1..n]$, where $D[i]$ = the shortest distance between s and i . Initially, $D[s] = 0$.
- Shortest paths: $Parent[1..n]$. Initially, $Parent[s] = 0$.
- $nearest[1..n]$, where

$$nearest[i] = \begin{cases} 0 & \text{if } i \in V' \\ \text{the node } x \text{ in } V' \text{ that} \\ \quad \text{minimizes } D[x] + length[x, i], & \text{if } i \notin V' \end{cases}$$

Complexity: $O(n^2)$