

# CSE 680 Homework 1

Due Wednesday, January 13 by class time

1. For each of the following functions  $f_i(n)$ , express its asymptotic complexity in the simplest form.

(a)  $f_a(n) = 4^{\log_2(n^2)} = \Theta(2^x)$ , where  $x = \underline{\hspace{2cm}}$ .

(b)  $f_b(n) = \sqrt{5n + 30} \cdot (\log_2(n^4 + 12n^2) + 7) = \Theta(\hspace{2cm})$

(c)  $f_c(n) = \log_2(n^3 - 6n) + \sqrt{32n + 680} = \Theta(\hspace{2cm})$

(d)  $f_d(n) = (4n^3 + n)^{0.1} \cdot (3n + 8)^{0.4} + \log_2(n^9 + 10n^2) = \Theta(\hspace{2cm})$

(e)

$$f_e(n) = \sum_{i=1}^{\log n} \frac{1}{i} = \Theta(\hspace{2cm})$$

2. Prove that  $\sqrt{5n + 30} \cdot \log_2(n^4 + 12n^2) = O(\sqrt{n} \cdot \log_2 n)$ .
3. Prove that if  $f_1(n) = \Omega(g_1(n))$  and  $f_2(n) = \Omega(g_2(n))$  then  $f_1(n) + f_2(n) = \Omega(\max(g_1(n), g_2(n)))$ .
4. Disprove the following statement by giving a counterexample: If  $f(n) = \Theta(n)$ , then  $f(n)$  is asymptotically monotonically non-decreasing (i.e.,  $f(n) \leq f(n + 1)$  for all sufficiently large  $n$ ). Note that  $f(n)$  is a function defined for all positive integers  $n$ .
5. Disprove the following statement by giving a counterexample: If  $f(n) \in O(g(n))$ , then  $O(f(n)) = O(g(n))$ . (Here we use the “more abstract” definition of big- $O$ .)