CSE541 Homework 5

Due Wednesday, May 7 at class time.

1. (a) Use the central difference formula with $h = 1.0$ to numerically approximate the derivative of $\sin(\cos(x))$ at $x = 2$;
   (b) Use Richardson's extrapolation to approximate the derivative of $\sin(\cos(x))$ at $x = 2$ starting with $h = 1.0$ and using four rows in the Richardson extrapolation table (i.e., compute up to $D(3, 3)$.) Give the complete table.
   (c) Compare with the exact value given by analytically differentiating $\sin(\cos(x))$.

2. (a) Apply a single formula to numerically approximate the second derivative of $\sin(\cos(x))$ (without any analytical differentiation) at $x = 2.0$ using $h = 1.0$;
   (b) Use Richardson's extrapolation to approximate the second derivative of $\sin(\cos(x))$ at $x = 2$ starting with $h = 1.0$ and using two rows in the Richardson extrapolation table (i.e., compute up to $D(1, 1)$.)
   (c) Analytically derive the second derivative and compare.

3. Derive the approximation formula

$$f'(x) \approx \frac{1}{2h}\left[4f(x+h) - 3f(x) - f(x+2h)\right]$$

   and show that its error term is $O(h^2)$. (Use the Taylor series expansion for $f(x + h)$ and $f(x + 2h)$.)

4. Assume that we are only given $f(x+h)$, $f(x)$ and $f(x-2h)$, and wish to approximate $f''(x)$. Prove that

$$f''(x) \approx \frac{1}{3h^2}\left[2f(x+h) - 3f(x) + f(x-2h)\right].$$

   (Use the Taylor series expansion for $f(x + h)$ and $f(x - 2h)$.)