LINE DIRECTION MATTERS: AN ARGUMENT FOR THE USE OF PRINCIPAL DIRECTIONS IN 3D LINE DRAWINGS

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ABSTRACT

While many factors contribute to shape perception, psychological research indicates that the direction of lines on the surface may have an important influence. This is especially the case when other techniques (shading, silhouetting) do not present sufficient shape information. The psychology literature suggests that lines in the principal directions of curvature may communicate surface shape better than lines in other directions. Moreover, principal directions have the quality of geometric invariance so line directions are based on the surface geometry and are viewpoint and light source independent, and the lines do not move above over the surface during animation unless desired. In this work we describe principal direction line drawings which show the flow of curvature over the surface. The technique is presented for arbitrary surfaces represented by either 3D volume data or a polygonal surface mesh. The latter format is common in the field of computer graphics yet thus far has not been widely used for principal direction estimation. The methods offered in this paper can be used alone or in conjunction with other NPR techniques to improve artistic 3D renderings of arbitrary surfaces.

Keywords: non-photorealistic rendering, principal direction line drawings, line direction, line drawings, geometrically invariant line drawings.

1 INTRODUCTION

Amongst the varied goals of artistic Non-Photorealistic Rendering (NPR) is the pursuit of *perceptually efficient* images. A perceptually efficient visual representation emphasizes important features and minimizes extraneous detail and is essential for making comprehensible artistic images. Computer-generated line drawings are a particularly effective form of NPR since lines' features (length, width, intensity, density, quality, direction, etc.) can be combined to create shaded, textured, and expressive images which capture the essence of the form of an object. In the field of computer-generated line drawing, 3D representations of curved surfaces generally focus on the silhouette edges, disregarding large amounts of interior curvature information. These depictions often rely on either previous knowledge of the surface or the use of motion (movement of the surface, viewpoint, or light source). In this work we explore a 3D line drawing technique which is independent of the surface's orientation, the viewpoint, or the light source. In particular, we examine line direction and use this paper to raise the question: Does line direction matter?

We argue that line direction does matter, and suggest the use of the principal directions of curvature for directing lines to improve the depiction of surface shape in artistic line drawings. The advantages of principal directions (see Appendix A for a mathematical definition) are that they are geometrically-invariant, highlight the most direct path on a surface between two points, indicate the directions of the curvature extrema at any point, and have been suggested by psychologists as the preferred interpretation for making surface shape judgments.

The importance of geometric invariance should not be underestimated. Geometrically-invariant cues are based on properties of the surface geometry and are by definition viewpoint and light source independent. While shading and silhouetting provide substantial shape information, valuable curvature information can be lost in shadows or the interior of the surface. Furthermore, viewpoint dependent lines may move around in a distracting manner during motion or animation. Geometric invariance does not imply that lines must be rigidly "pasted" onto the surface during animation. If line movement is desired, the geometrically-invariant vector field can help guide more fluid movement over the surface. Combining geometrically-invariant cues with shading or silhouetting can be especially powerful. Geometrically-invariant line attributes such as color and density can be manipulated with respect to viewpoint or light source [7].

Despite the promise for principal directions, their full potential in NPR has yet to be realized. The reasons perhaps may be related to the difficulties in estimating an accurate, smoothly continuous vector field of principal directions. The problem is most challenging for polygonal surface meshes, a particularly common data format for arbitrary 3D surfaces. Additionally, principal direction line drawings must address the complex issues of creating uniformly distributed, non-intersecting, long smooth lines which gracefully traverse umbilics, planar regions, and transitions of directional dominance. Here we examine both 3D volume datasets and polygonal surface meshes, and suggest some techniques for line tracing.

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The main contribution of this work is to show that for a 3D line drawing, line direction can matter and principal direction line drawings can be used to better convey surface shape. In the next section we motivate the importance of line direction with psychological evidence. We follow with related work in computer-generated 3D line drawing. In section four, we provide a brief overview of principal direction estimation techniques. Section five shows the effects of line direction and section six presents techniques for principal direction line drawings. In the final section we draw some conclusions and discuss areas of future work.

2 PSYCHOLOGICAL EVIDENCE FOR THE IMPORTANCE OF LINE DIRECTION

The psychology literature gives us a sense of how the human visual system perceives images and is an essential reference for making perceptually efficient renderings. Early research asserted that humans can use surface markings, or texture, to perceive surface orientation. Gibson [8] was amongst the first to emphasize the significance of texture cues for shape and depth perception. He was able to show convincingly that observers could reliably interpret the slant of the planar surface by the cues provided by the projection distortion of the texture patterns.

Of relevance to this work is the open question of whether anisotropic (directed) textures are as suitable for conveying shape information as isotropic (undirected) textures. Interrante [12] was unable to show an effect of texture type in shape perception under conditions of stereo and motion for various plausible isotropic and anisotropic textures for transparent surfaces, including grids and principal direction textures. Yet Cumming et al. [3] found an indicative effect of texture type for stereoscopic shape perception between a plausible and unlikely texture. While shape-from-texture research often makes assumptions of isotropy or homogeneity, Knill [16] hypothesized that there are different modes to visually perceive isotropic and anisotropic textures.

While the question of effects of isotropic versus anisotropic texture still remains open, it is evident that when anisotropic surface markings are dependent on surface geometry, surface depth and orientation perception is improved. Knill [16] found that in an anisotropic texture processing mode, the curvature of geodesic surface markings determines perception of local surface orientation. The experiments of Johnston et al. [14] showed that stereoscopic depth perception of curved surfaces with texture which provided a good indication of surface geometry was superior to random dot textures. Stevens [24] was among the first to suggest that humans can make surface shape judgments by assuming that surface contours (lines on the surface) are aligned with the principal directions of curvature. In later work Stevens and Brookes [23] demonstrated that principal direction surface contours are also good indications of relative surface slant. More recently, Mamassian and Landy [17] found that surface shape judgments are biased by the assumption that surface contours are aligned with the principal directions. From the above literature, it is reasonable to believe that surface shape and depth perception may be generally aided by textures, and also by anisotropic textures based on surface geometry, particularly lines aligned with the principal directions

3 RELATED WORK

Computer-generated 3D line drawings borrow from centuries of artists' techniques and have recently received significant attention in the NPR community. Winkenbach and Salesin used stroke textures to create depth and shape in line drawings of parametric surfaces [26]. Markosian et al. emphasized the silhouette edges for viewpoint-dependent images of arbitrary 3D surfaces [18]. Curtis used 3D models to generate loose and artistic sketches and animations [4]. Elber rendered geometrically-invariant line drawings and textures of parametric and implicit surfaces [6].

Principal directions have been suggested [11,26] and approached [2,6] in line drawings. In [26,6], lines were traced along the parametric lines of parametric surfaces, which sometimes coincided with the principal directions. Saito and Takahashi [20] rendered line drawings lines of parametric surfaces along geodesic lines. Interrante et al. [11] used 3D principal direction textures to illustrate surface shape in volume data. However, none of these works addressed the challenge of estimating the principal directions from arbitrary surfaces (particularly polygonal surface mesh formats) nor that of tracing long strokes in one direction (rather than cross-hatching) through umbilics, planar regions, and areas of changing directional dominance. This work is based upon a preliminary sketch by Girshick and Interrante [9].

4 PRINCIPAL DIRECTION ESTIMATION

For data of any format, the first step towards a principal direction line drawing is to estimate the principal direction vector field, comprised of the principal directions at a set of points on the surface. There are a variety of methods for estimating principal directions, each with its various strengths and weaknesses, however a full discussion of the computational details is not in the scope of this paper. Do Carmo outlines analytic calculations of principal directions for parametric surfaces in [5]. For iso-intensity surfaces in 3D volume data, Monga et al. used the Hessian of the 3D data to compute the principal directions [19]. Interrante et al. used a similar technique based on Gaussian-

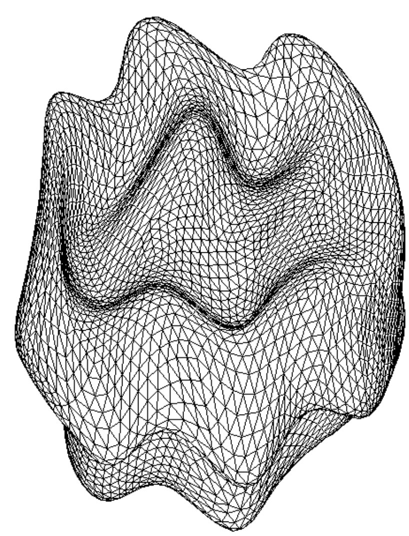


Figure 1 Polygonal surface mesh of arbitrary 3D "blob".

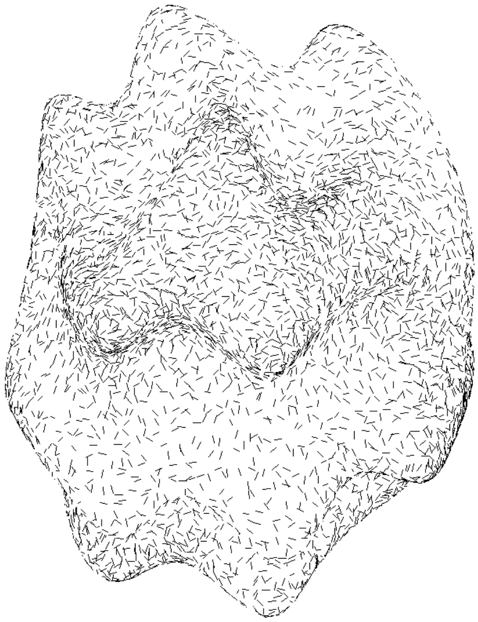


Figure 2 Random vector field of object in figure 1.

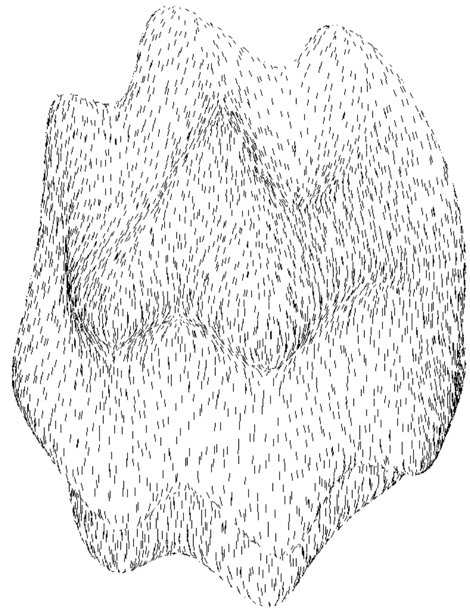


Figure 3 Uniform (vertical) vector field of object in figure 1.

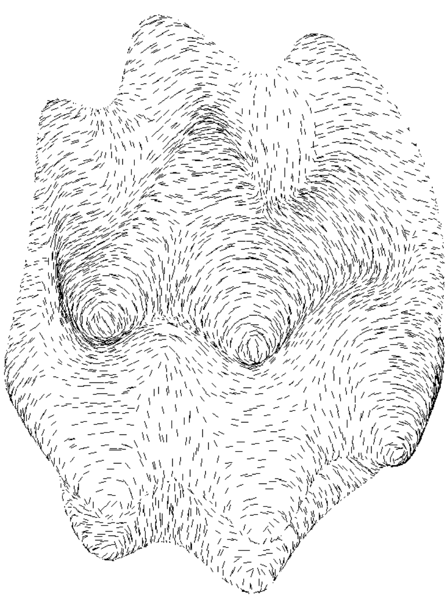


Figure 4 First principal direction vector field of object in figure 1.

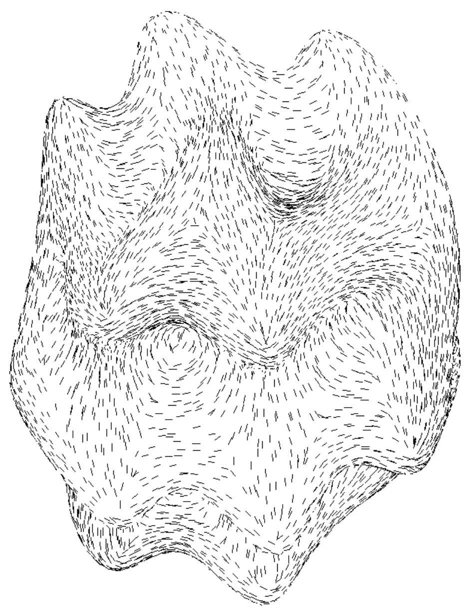
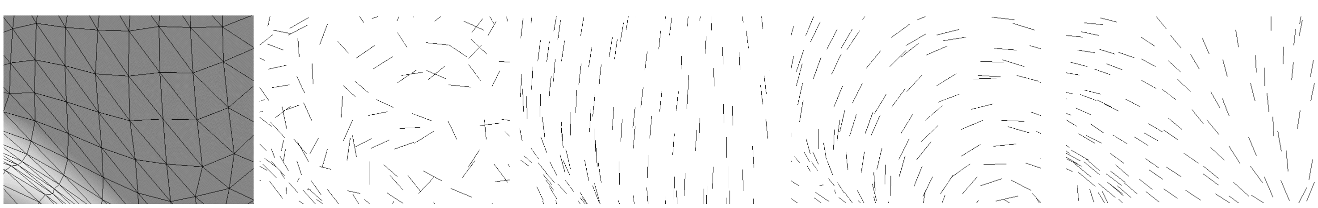


Figure 5 Second principal direction vector field of object in figure 1.



6a Shaded surface mesh **6b** Random vector field **6c** Uniform vector field **6d** First principal direction vector field **6e** Second principle direction vector field

Figure 6 Close-ups of the same region of the object in figure 1.

weighted finite-differencing [12]. We used this approach for the volume datasets in this paper.

As of yet there is no reliable standard technique for locally estimating principal directions from a polygonal surface mesh. Samson and Mallet [21] fit cubic patches to the local neighborhood around a vertex, using the vertex's normal and neighboring normals, and then compute the partial derivatives to obtain principal directions. Hamann [10] employs a similar approach except uses quadratic patches and relies solely on deviation from a vertex's tangent plane without using neighboring vertex normals. Joshi et al. provide good examples of this approach in [15]. Chen and Schmitt [1] and Taubin [25] avoid explicitly describing surface patches but instead construct a quadratic form at each vertex. In [25] the quadratic form represents an orthonormal basis whose eigenvectors are the principal directions. The principal curvatures are the directional curvatures in the principal directions. For the polygonal surface meshes in this work, we use variations of both Hamann's and Taubin's methods, with similar results. The accuracy of both is highly dependent on the symmetry of the local surface geometry and is an area of current work.

5 EFFECTS OF LINE DIRECTION

The significance of line direction for a line drawing is perhaps best illustrated visually with the underlying vector field. As will be explained in the next section, a line drawing can be rendered by tracing strokes which follow the flow of a vector field [22]. Figures 1–5 show various vector fields on the same arbitrary "blob" dataset, shown as a polygonal surface mesh in figure 1. A 3D volume dataset would produce similar results. The vector field is illustrated by projecting the field direction at each vertex of the underlying mesh onto the tangent plane at that point. The random vector field in figure 2 and the uniform vector field in figure 3 convey surface shape only through texture compression, which provides hints of the silhouette edges, but not through the use of line direction. When the silhouette edges are not visible, as in the close-ups in figures 6b and 6c, the surface shape is largely ambiguous.

Figures 4 and 5 show first and second principal direction vector field respectively. Compared to figures 2 and 3, these vector fields appear to better convey local surface orientation, including ridges and valleys, subtle surface undulations, changes in curvature, and interior silhouette edges. Figure 6 shows the close-ups of the vector fields in the absence of silhouette edges. When comparing the four close-ups in figures 6b through 6e,

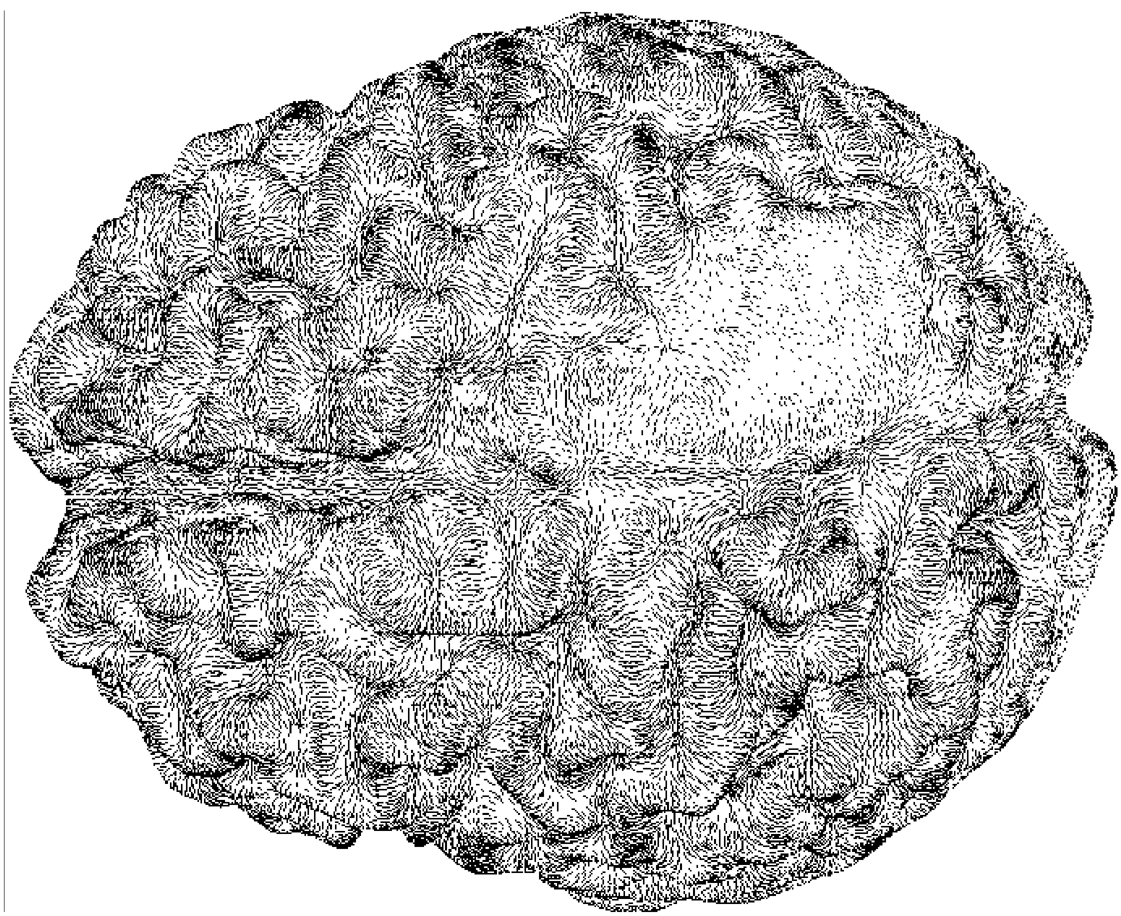


Figure 7 First principal direction vector field of a brain represented by a polygonal surface mesh. Data source: Ron Kikinis, Harvard Medical School.

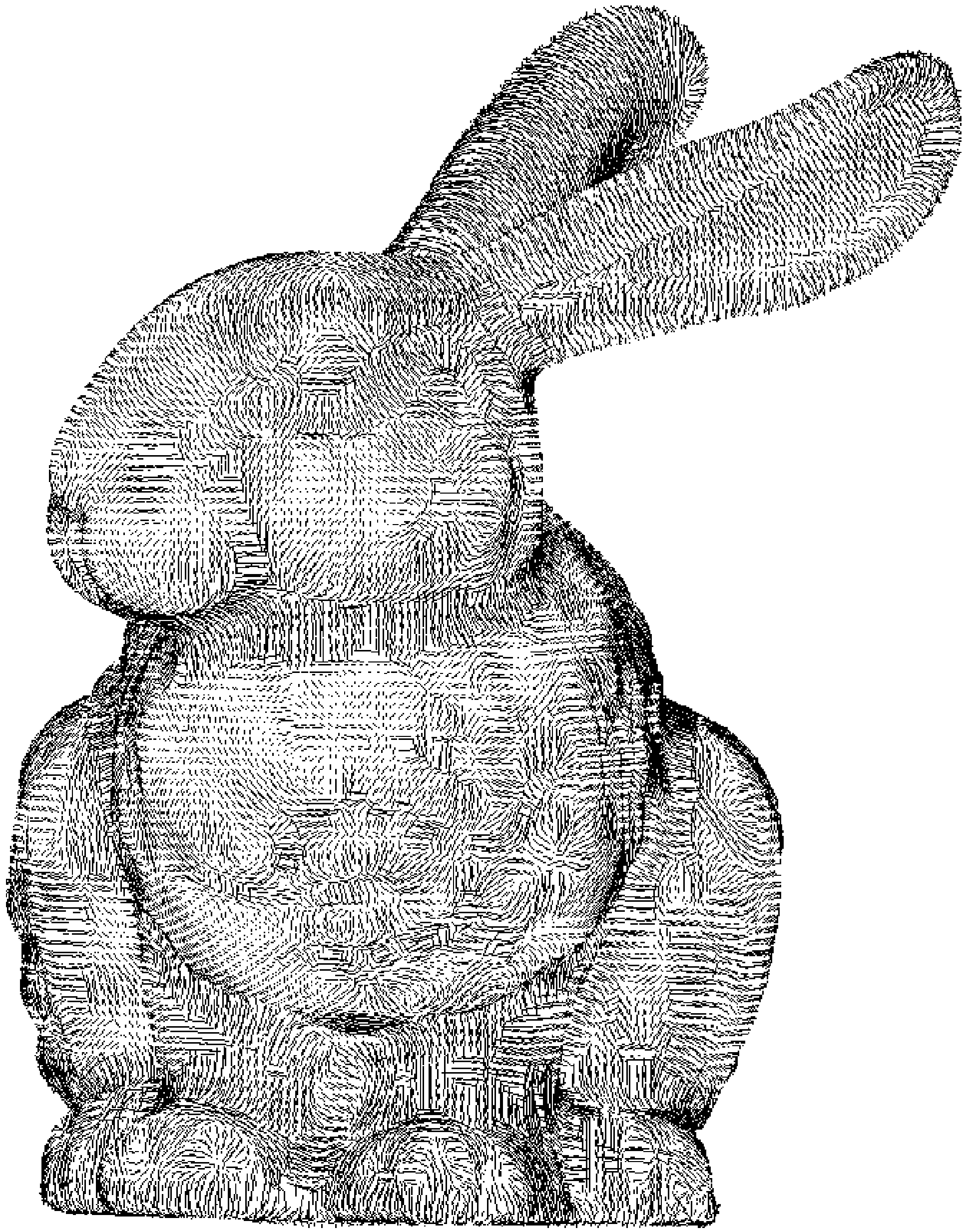


Figure 8 First principal direction vector field of a bunny represented by a polygonal surface mesh. Data source: Stanford University Computer Graphics Lab.

it seems to be easier to judge the surface shape from principal direction vector fields than the random and uniform vector fields. Figures 7 and 8 provide more examples of first principal direction vector fields on more complex surfaces. One can predict the difficulty in perceiving the surface shape if these figure used random or uniform vector fields.

6 PRINCIPAL DIRECTION LINE DRAWINGS

Principal direction line drawings illustrate the flow through the principal direction vector fields described in the previous section. In this section we describe the details for both 3D volume data and polygonal surface meshes. For 3D volume data, the vector field is a 3D volume and the strokes are traced through the volume. For polygonal surface data, the vector field lies on the explicitly defined surface mesh and the strokes must be drawn on the surface.

6.1 Principal Direction Line Drawings of 3D Volume Datasets

Figures 9 and 10 show different styles of principal direction line drawings of the same human pelvis CT volume dataset. Both figures underwent the same preprocessing stage. Initially a first principal direction volume vector field is generated using the technique described in section four. Then a sparse set of strokes

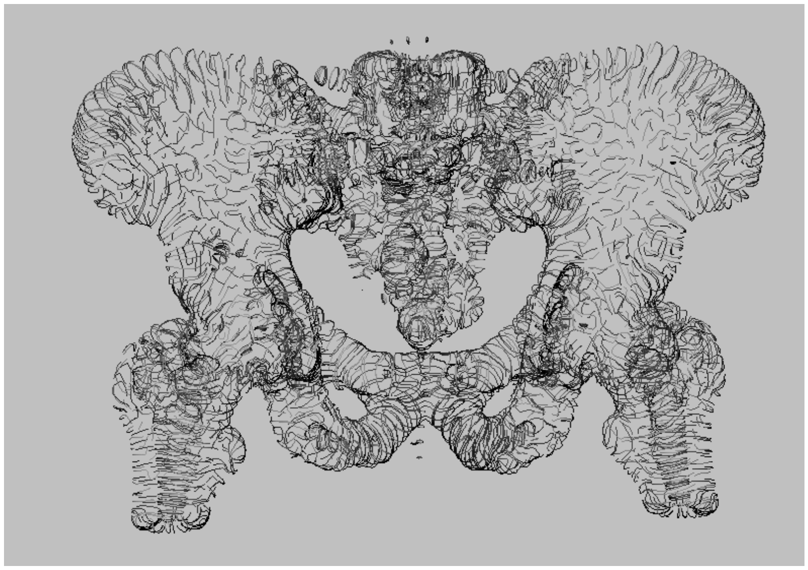


Figure 9 Principal direction line drawing (with shading and without hidden line removal) of a bone/soft tissue boundary iso-intensity surface in a CT 3D volume dataset of a human pelvis.

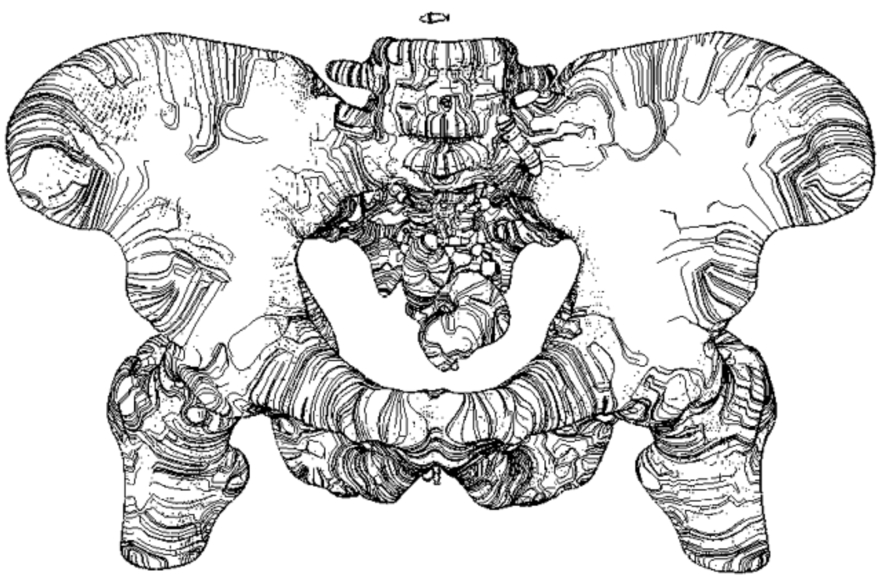


Figure 10 Principal direction line drawing with silhouette edges and hidden line removal of the volume dataset in figure 8.

is traced through the vector field, each stroke originating from a point near the surface which is not too close to neighboring starting points, such that the set has the approximate distribution of a Poisson disk.

In figure 9, the strokes represent individual streamlines [22] through the vector field. The lines are shaded according to the surface normal direction indicated by the gray level gradient in the volume data, but hidden line removal has not been done. The result is especially powerful during animation, when the geometrically-invariant lines "stick" to the surface.

In figure 10 we attempted to create a freer sketch of the volume data set, using hidden line removal, including silhouettes and selecting only a subset of possible strokes. Because it is viewpoint-dependent, by definition it is not geometrically-invariant. However the lines are still directed in the principal directions and defined based on the geometry of the surface, so the static 2D image should provide the same visual cues to the surface shape, at least near the silhouette edges. The subset of lines to render was selected with a preference towards placing lines in areas of higher curvature lines and near silhouette edges. Line length is proportional to the magnitude of the first principal curvature at the start point.

6.2 Principal Direction Line Drawings of Polygonal Surface Meshes

The main steps in creating a principal direction line drawing from a polygonal surface mesh are estimating a smoothly continuous principal direction vector field and tracing evenly spaced strokes which follow the flow of the vector field. The steps are described separately below, but for efficiency they can be done simultaneously.

6.2.1 Creating a continuous principal direction vector field

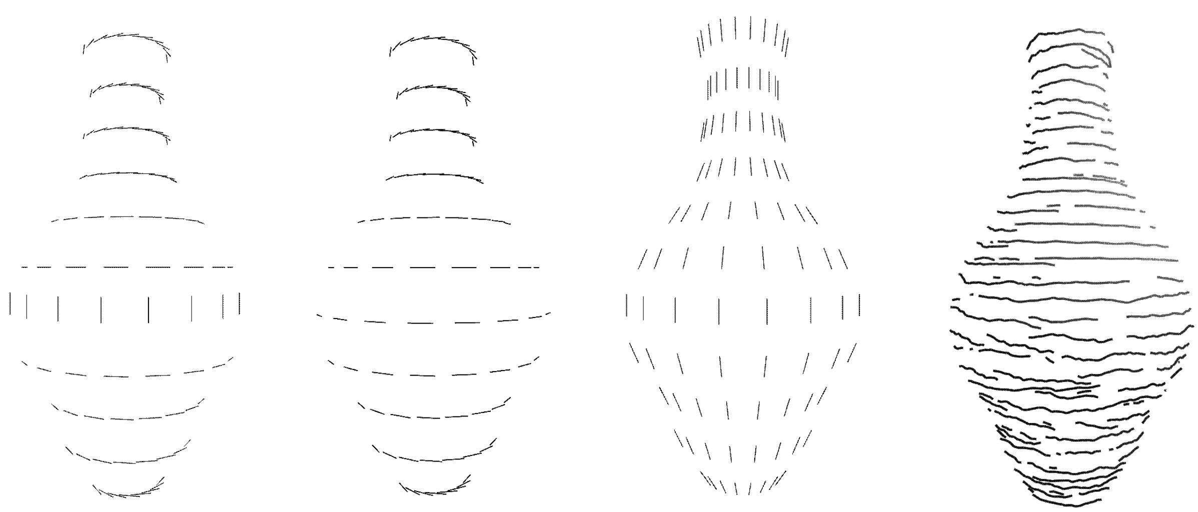
At any point on a 3D surface, each of the orthogonal first and second principal directions have a positive and negative direction. Thus there are four possible directions for the vector field at each point. Ideally we would always choose the first principal direction (either positive or negative). However, in regions close to umbilics and planes, where curvature is almost similar in all directions, the first and second principal directions may suddenly switch places causing a flip of up to 90 degrees, resulting in a sudden disruption of flow. Figure 11a demonstrates this for a simple vase mesh. The first principal direction field is continuous except around the girth of the vase where it is *almost* spherical and the curvature is slightly greater vertically than horizontally. In this case, a *continuous* principal direction line drawing minimizes distracting details and is more aesthetically pleasing than a *first* principal direction line drawing. The continuous vector field is created by first choosing an arbitrary reference vector. In the example of figure 11, the choice of reference vector can lead to only two possible outcomes, but in a more complex dataset it might be advantageous to choose a meaningful starting reference vector. Next, for each vertex, the direction which is closest to the reference vector is chosen. The reference vector is updated to reflect the choice. Figures 11b and 11c show the two possible continuous principal direction vector fields for this dataset. The principal direction line drawing corresponding to 11b is shown in 11d. This approach for creating continuous vector field works well for surface regions with well-defined principal directions. However, at true umbilics, where normal curvature is the same in all directions, and on planes, where normal curvature is zero in all directions, the principal directions are undefined. For these regions, we interpolate between neighboring well-defined regions of the vector field. Even still, for a complex surface such shown in figures 1 and 8, regions may occur where it is necessary to make an abrupt switch in line direction. A possible technique for gracefully transitioning between line directions is to minimally employ cross-hatching using both the first and second principal direction fields combined. However we do not advocate the general use of crosshairs such as in figure 12, as the inelegant crosses can become distracting and muddle the flow of curvature.

6.2.2 Tracing strokes through the vector field on a polygonal surface

The objective of this step is to obtain an approximately uniformly-distributed set of non-intersecting long curved lines, which lie on the surface. The streamline tracing technique of Jobard and Lefer [13] is extended from 2D images to 3D surfaces to generate evenly-spaced non-intersecting lines. The curvature of each line is achieved by continually redirecting it as it traverses the changing vector field.

Each stroke is composed of a set of control points. The criterion for each valid control point is that it lies at a minimum distance threshold from all existing strokes. The first stroke starting point is random, and the remaining stroke starting points are chosen to be as close as possible to existing points without breaking the minimum distance threshold.

The direction of the stroke is updated at frequent distance intervals as well as when a stroke crosses a polygon boundary. The stroke's direction at any given point on a polygon is



11a First principal direction vector field.

11b Continuous vector field of greatest overall curvature.

11c Continuous vector field of less overall curvature.

11d Continuous principal direction line drawing for 11b. Shading and slight randomness added to strokes for artistic effect.

Figure 11 Various principal direction vector fields and principal direction line drawing of a simple vase.

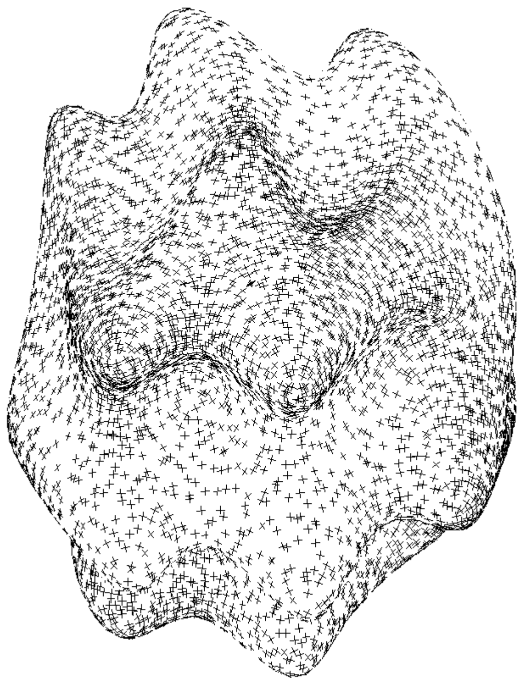


Figure 12 First and second principal direction vector field of the object in figure 1.

determined by trilinearly interpolating the principal directions of the polygon's vertices. Strokes are terminated if they approach the minimum distance threshold. This process is shown in figure 13. To avoid the cost of calculating an implicit surface, each segment of a stroke is projected onto the polygonal surface mesh. Provided a sufficiently fine mesh, this approximation is worth the savings in computation.

Regions of opposing force occur when neighboring principal directions point in opposing directions. These vector field discontinuities crop up near umbilics and planar. The current approach is to terminate strokes when this happens.

The result of this technique is shown in figure 14, with some randomness added for wiggly lines. A more artistic image might be achieved by varying the line density according to the light source, and adding silhouette lines.

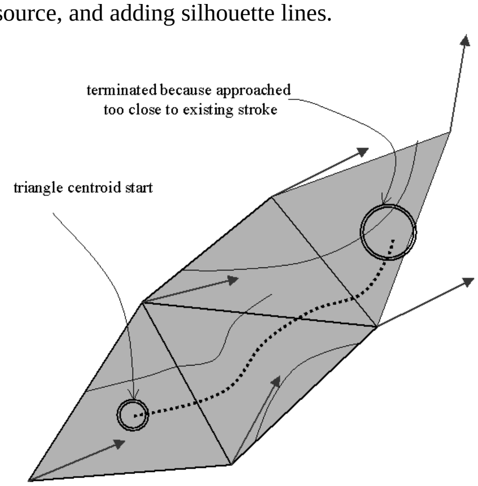


Figure 13 Stroke tracing through a principal direction vector field on a polygonal surface mesh. (image for illustrative purposes only).

Figure 14 Principal direction line drawing of pears, represented by triangular surface meshes. Hidden line removal was used, and slight random noise was added to the stroke tracing process.

6.2.3 Rendering

The rendering of the line drawing is straightforward. A stroke is a set of control points which can be rendered as either a simple polyline or spline. Our approximations were fine enough to use anti-aliased polylines with no perceivable difference over splines.

7 CONCLUSIONS AND FUTURE WORK

The most troublesome areas in obtaining a continuous principal direction line drawing are those where the principal directions are undefined and in regions of opposing force. In the first case, the current interpolation technique works well if the unknown regions are small and bordered by more well-defined principal directions, but fails for larger areas and is a topic of future work. In the latter case, we would like to eventually gracefully merge strokes from neighboring regions of opposing principal directions, possibly with subtle cross-hatching, instead of terminating them.

This work outlined the approach for both 3D volume datasets and polygonal surface. Principal direction line drawings for parametric surfaces can follow a similar approach. For polygonal surface meshes, we found the existing principal direction estimation techniques to be insufficiently accurate for asymmetric local mesh geometries. We are currently working on their improvement which is of great relevance to principal direction line drawings.

We also found that principal direction vector fields work well, in conjunction with silhouette lines or shading, as "short stroke" principal direction line drawings. An example of this is shown in figure 15. Future work includes extending these lines with the line drawing technique described above, using density variations for shading.

This work poses the important question of whether line direction matters for creating a perceptually efficient line drawing. We have provided compelling psychological evidence and visual examples to believe that line direction affects surface shape perception. In particular, the principal directions of curvature appear to be more effective than non-principal directions at conveying surface shape. Principal direction lines on a surface have the advantage that they show the path of greatest curvature and are geometrically-invariant, so they appear the same from all

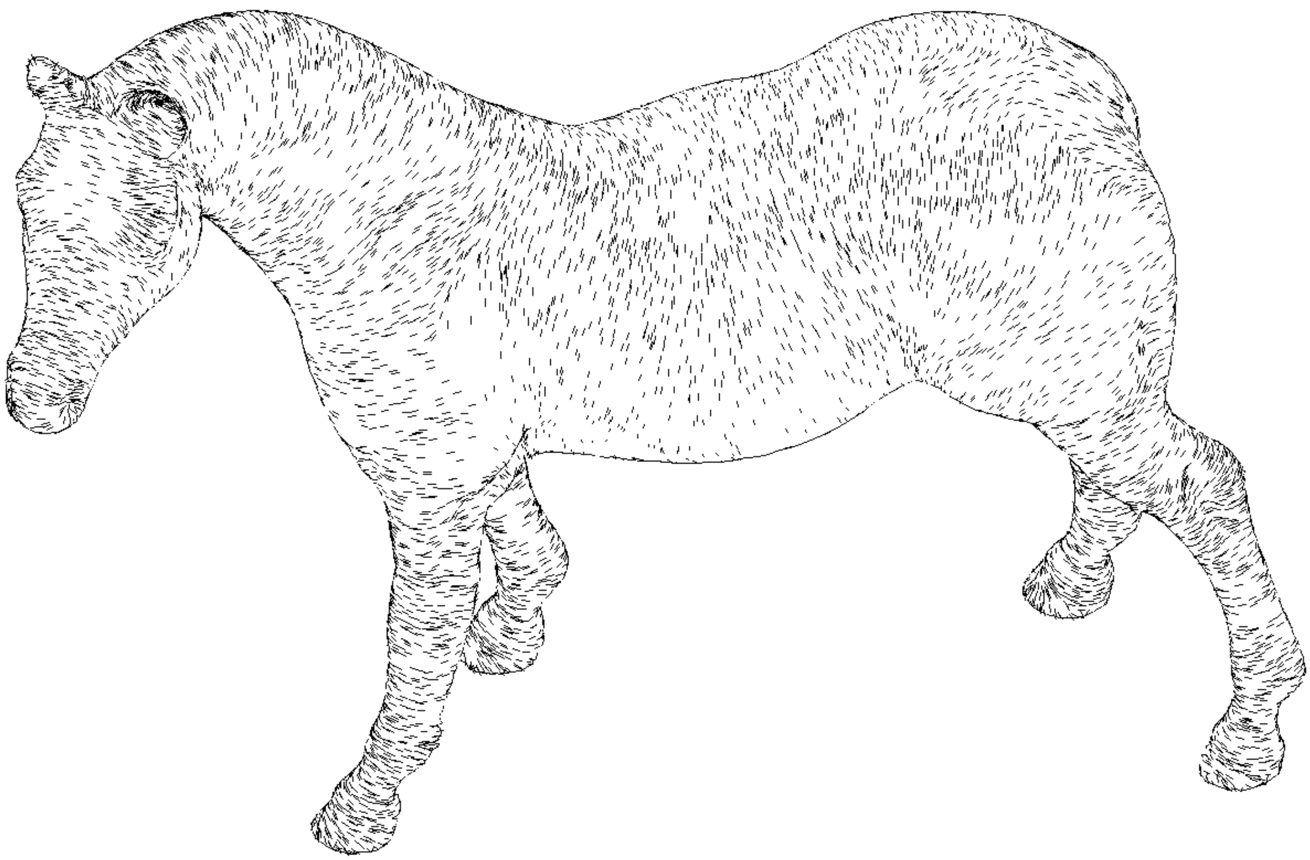


Figure 15 First principal direction vector field and silhouette lines of a horse dataset, courtesy of Cyberware, Inc.

viewpoints and do not shift during animation. Principal direction line drawings are well-suited for showing the subtle undulations of an arbitrary, smoothly curved surface in 3D, especially when silhouette edges are not visible. They can be used alone or in conjunction with other graphics techniques such as shading and drawing silhouette edges. One intention of this work is to serve as a reminder that perceptually efficient images are an important part of artistic NPR. We also wish to inspire more perceptual studies of the effectiveness of principal direction line drawings.

8 ACKNOWLEDGMENTS

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9 APPENDIX A: DEFINITION OF PRINCIPAL DIRECTIONS OF CURVATURE

The normal curvature at p in a given direction T will be referred to as the directional curvature $\kappa_p(T)$. The *first principal direction*, T_1, is the direction of the maximum magnitude of normal curvature, called the *first principal curvature* (κ_p^1). The *second principal direction*, T_2, is orthogonal to the first, and is the direction of the other curvature extreme, called the *second principal curvature* (κ_p^2). For elliptic surface patches (with positive Gaussian curvature) the second principal direction is the direction which the surface is most nearly flat. For hyperbolic, (saddle-shaped) patches (with negative Gaussian curvature), the second principal direction is the direction of the lesser of the two extrema. The two principal directions T_1 and T_2 are orthogonal and lie in the tangent plane at the point p, creating an orthonormal basis with the normal vector N at p. Figure 14 shows an example of the orthonormal basis on a hyperbolic surface patch. The product of the two principal curvatures equals the Gaussian curvature, $K=\kappa_p^1 \bullet \kappa_p^2$.

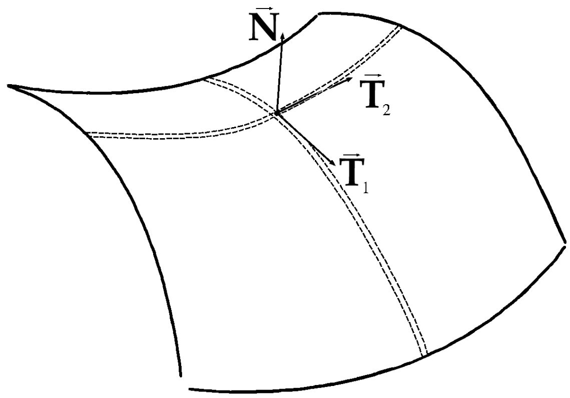

Figure 16 Orthonormal basis formed by normal and two principal directions and curvature strips in the principal directions at a point on a hyperbolic patch.

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