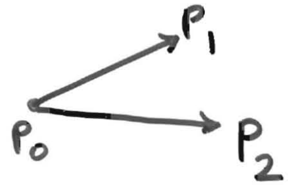
Computational Geometry

First we see some basic operations on line segments that we will use later.

Let P_0P_1 and P_0P_2 be two line segments.



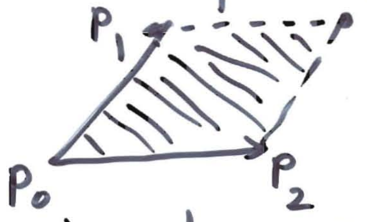
- Determine if P_0P_2 is clockwise turn from P_0P_1:

  Compute $\text{Det}\begin{pmatrix} x_1-x_0 & x_2-x_0 \\ y_1-y_0 & y_2-y_0 \end{pmatrix}$.

  if the Det is positive, then P_0P_2 is clockwise turn from P_0P_1; if zero, then $P_0P_1P_2$ are collinear; if negative, then anticlockwise.

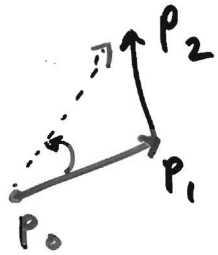
This is coming from the cross product

$$(P_1-P_0) \times (P_2-P_0)$$



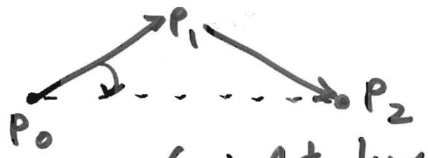
The determinant is the signed area.

• Determine if



(left-turn)

or

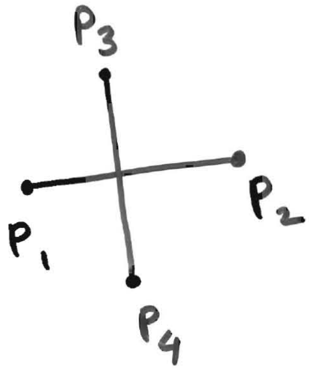


(right-turn)

Again, look at $(P_2 - P_0) \times (P_1 - P_0)$ by taking the $\text{Det}\begin{pmatrix} x_2 - x_0, & x_1 - x_0 \\ y_2 - y_0 & y_1 - y_0 \end{pmatrix}$.

if it is negative, then left-turn, and so on.

• Determine if $\overline{P_1P_2}$ & $\overline{P_3P_4}$ intersect.



Let Direction routine computes relative orientations.
and On-Segment " " & collinearity.

Segment-Intersect (P_1, P_2, P_3, P_4)

$d_1 := \text{Direction}(P_3, P_4, P_1)$

$d_2 := \text{Direction}(P_3, P_4, P_2)$

$d_3 := \text{Direction}(P_1, P_2, P_3)$

$d_4 := \text{Direction}(P_1, P_2, P_4)$

if $((d_1 > 0$ and $d_2 < 0)$ or $(d_1 < 0$ and $d_2 > 0))$

and

$((d_3 > 0$ and $d_4 < 0)$ or $(d_3 < 0$ and $d_4 > 0))$

then return true

elseif $d_1 = 0$ and On-Segment (P_3, P_4, P_1)

then return true

elseif $d_2 = 0$ and On-segment (P_3, P_4, P_2)

then return true

elseif $d_3 = 0$ and On-segment (P_1, P_2, P_3)

then return true

elseif $d_4 = 0$ and On-segment (P_1, P_2, P_4)

then return true

else return False.

Direction (P_i, P_j, P_k)

return $(P_k - P_i) \times (P_j - P_i)$.

On-Segment (P_i, P_j, P_k)

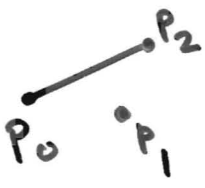
if $\min(x_i, x_j) \leq x_k \leq \max(x_i, x_j)$

and $\min(y_i, y_j) \leq y_k \leq \max(y_i, y_j)$

then return true

else " false.

Determine which side of the line segment $\overline{P_0P_2}$ P_1 lies.



We can again take $\mathrm{Det}\begin{vmatrix} x_1-x_0 & x_2-x_0 \\ y_1-y_0 & y_2-y_0 \end{vmatrix}$

if this is positive then P_1 lies below the oriented line $\overrightarrow{P_0P_2}$ and so on.

An observation:

$$\mathrm{Det}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_0 & y_0 & 1 \end{vmatrix} = \mathrm{Det}\begin{vmatrix} x_1-x_0 & y_1-y_0 & 0 \\ x_2-x_0 & y_2-y_0 & 0 \\ x_0 & x_0 & 1 \end{vmatrix}$$

$$= \mathrm{Det}\begin{vmatrix} x_1-x_0 & y_1-y_0 \\ x_2-x_0 & y_2-y_0 \end{vmatrix}$$

$$= \mathrm{Det}\begin{vmatrix} x_1-x_0 & x_2-x_0 \\ y_1-y_0 & y_2-y_0 \end{vmatrix}.$$