Randomized Max-3-CNF-SAT (Cont...)

The previous randomized algorithm gave an expected 7/8-approximation of the maximum.

We can turn statement to something like:

Algorithm produces at least 7/8-approx with high probability.

- First observe that since $E[Y] = \frac{7}{8}m$, there exists an assignment of the variables that makes at least $\frac{7}{8}$ of the total clauses satisfied.

- Can we find such an assignment with high probability?
  Or, at least a little worse approximation, say $\frac{3}{4}$?

- We use the well known Markov's inequality.

Let X be a nonnegative random variable with a known expectation and a positive number $\rho \in \mathbb{R}^+$, then

$$\Pr[X \geq \rho] \leq \frac{E[X]}{\rho}.$$

- To use the above, first consider the random variable

$$X_i = \begin{cases} 1 & \text{if clause } i \text{ is not satisfied} \\ 0 & \text{o.w.} \end{cases}$$

- We already know $\Pr(X_i) = \frac{1}{8}$
- $E[X_i] = \frac{1}{8}$
- $E[X] = \sum_{i=1}^{m} \frac{1}{8} = m\frac{1}{8}$, where $X = X_1 + \cdots + X_m$
- $\Pr[X \geq \frac{m}{4}] \leq \frac{m}{8} / \frac{m}{4} \leq \frac{1}{2}$.

- The above implies that

$$\Pr\left[Y < \frac{3}{4}m\right] \leq \frac{1}{2}$$

$$\Pr\left[Y \geq \frac{3}{4}m\right] > \frac{1}{2}$$

Therefore, the random assignment satisfies at least $\frac{3}{4}$ the clauses with prob. at least $\frac{1}{2}$.