Introduction to Algorithms
Recursion & Merge Sort

CSE 680
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Motivation

- For insertion sort (and other problems) as *n* doubles in size, the quadratic quadruples!
- Can we decrease *n*?
- What if we ***Divide*** the sort into smaller pieces?
- We can then solve those (***Conquer*** them).
- We need to be able to combine the pieces in a manner simpler than quadratic.

Divide and Conquer

- Divide (into two equal parts)
- Conquer (solve for each part separately)
- Combine separate solutions
- Merge sort
  - Divide into two equal parts
  - Sort each part using merge-sort (recursion!!!)
  - Merge two sorted subsequences

Merge Sort

```
MergeSort(A, left, right) {
  if (left < right) {
      mid = floor((left + right) / 2);
      MergeSort(A, left, mid);
      MergeSort(A, mid+1, right);
      Merge(A, left, mid, right);
  }
}

// Merge() takes two sorted subarrays of A and
// merges them into a single sorted subarray of A
//    (how long should this take?)
```

Merge Sort: Example

- Show MergeSort() running on the array

```
A = {10, 5, 7, 6, 1, 4, 8, 3, 2, 9};
```

Analysis of Merge Sort

```
Statement                                                   Effort
MergeSort(A, left, right) {                                 T(n)
   if (left < right) {                                      Θ(1)
      mid = floor((left + right) / 2);                      Θ(1)
      MergeSort(A, left, mid);                              T(n/2)
      MergeSort(A, mid+1, right);                           T(n/2)
      Merge(A, left, mid, right);                           Θ(n)
   }
}
```

- So T(n) = $\Theta(1)$ when n = 1, and
  $2T(n/2) + \Theta(n)$ when n > 1
- So what (more succinctly) is T(n)?

Recurrences

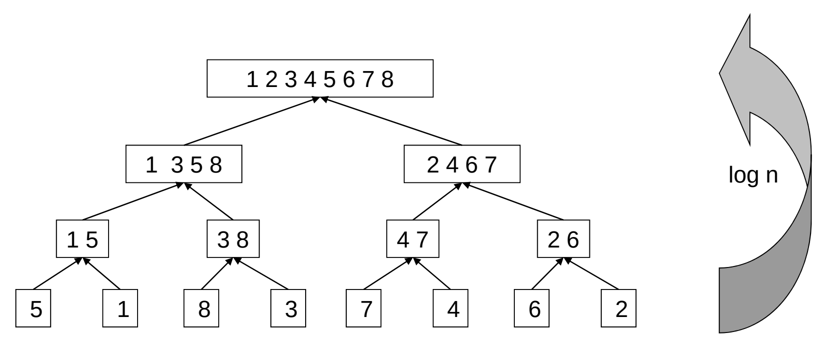
- The expression:

$$T(n) = \begin{cases} c & n = 1 \\ 2T\left(\frac{n}{2}\right) + cn & n > 1 \end{cases}$$

- is a *recurrence*.

Recurrence: an equation that describes a function in terms of its value on smaller functions

Recursion Tree



- n comparisons per level
- log n levels
- total runtime = n log n

Recurrence Examples

$$T(n) = \begin{cases} 0 & n = 0 \\ c + T(n-1) & n > 0 \end{cases}$$

Recurrence Examples

$$T(n) = \begin{cases} 0 & n = 0 \\ n + T(n-1) & n > 0 \end{cases}$$

Recurrence Examples

$$T(n) = \begin{cases} c & n = 1 \\ 2T\left(\frac{n}{2}\right) + c & n > 1 \end{cases}$$

Recurrence Examples

$$T(n) = \begin{cases} c & n = 1 \\ aT\left(\frac{n}{b}\right) + cn & n > 1 \end{cases}$$

Solving Recurrences

- Chapter 4 will look at several methods to solve these recursions:
  - Substitution method
  - Recursion-tree method
  - Master method